Population Growth, the Natural Rate of Interest, and Inflation

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Working Paper 03/2019**)
February, 2019

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February 10, 2019

Abstract

Population growth rates have fallen considerably in most developed countries. An important question for monetary policy is whether this has led to a fall in the natural rate of interest. In representative agent models, the response of the natural rate to a fertility shock crucially depends on the preference parameter determining how households weight generations of different size. Estimating a medium-scale model of the US-economy featuring fertility shocks, I find that declining population growth has lowered both the natural rate and inflation by about 0.4 percentage points in recent decades.

Keywords: Inflation, business cycles, monetary policy, natural rate of interest, demographic transition.

JEL Codes: D64, D91, E31, E32, E52, J11.

*I thank Mirko Wiederholt for his comments and his guidance. I also thank Ester Faia, Ctirad Slavik, Philipp Harms, the audience at the Money and Macro Brown Bag Seminar at Goethe University Frankfurt, and the participants at the ECB Forum on Central Banking 2016 in Sintra for their helpful comments. All remaining errors are mine.

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1 Introduction

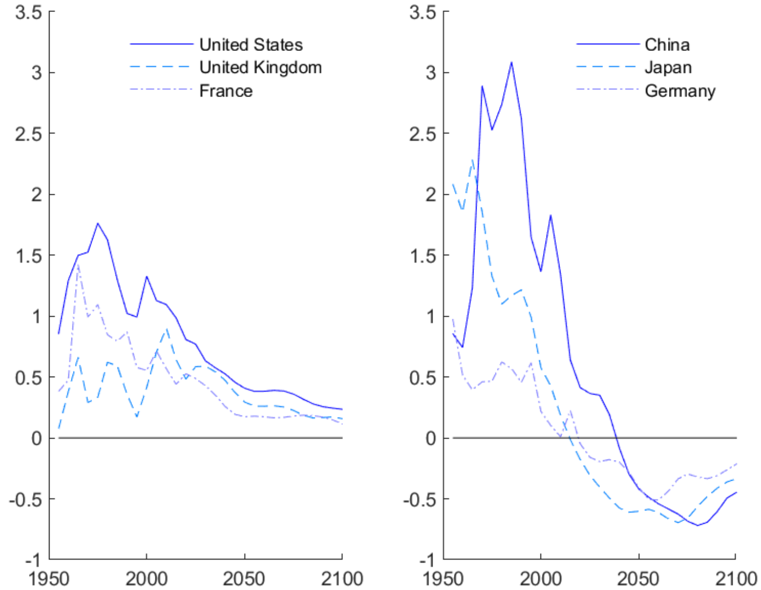
Over the last decades, population growth rates have fallen considerably across the world's largest economies (Figure 1). Current growth rates for the United States, the United Kingdom, France, Germany, Japan, and China are all below one percent. At the same time, inflation and interest rates have declined towards historically low levels. This raises the following question: How much of the observed fall in inflation and interest rates can be attributed to lower population growth? To address this question formally, I incorporate stochastic population growth in a standard business cycle model and estimate the macroeconomic effects of a fertility shock.

With a varying population size, the model-implied impulse responses depend on how households weight present and future generations. There are two polar cases. In the first case, households maximize total utility, i.e. per-person utility multiplied by the household size (*Benthamite* preferences). In the second case, households maximize utility per person, irrespective of the household size (*Millian* preferences). The preference specification influences the process of capital accumulation after a change in the population growth rate, and therefore the real interest rate that prevails or would prevail under flexible prices and wages.[1] This so called "natural rate of interest" is a key variable for monetary policy. Changes in the population growth rate lead to changes in the natural rate of interest in the Millian case, whereas no link between the two variables exists in the Benthamite case.

The reason is as follows. Consider a permanent fall in the population growth rate. Suppose that households do not change their saving behavior, i.e. they accumulate capital at the same rate as before, when population growth was higher. This leads to a reduction in the marginal product of capital, reflecting the increase in the capital-labor ratio. As a consequence, the return to capital falls with a lower population growth rate.[2] When the saving rate is constant, as in the Solow model, population growth and the natural rate are positively linked. In the Ramsey model, where saving is endogenous, this is not necessarily the case. With fewer workers joining the labor force, less investment is needed to maintain the same per-person capital stock. In the Benthamite case, households weight each generation by its size. Larger generations thus receive a larger weight, whereas smaller generations receive a smaller weight. In the long run, house-

[1]The term *natural rate of interest* goes back to Wicksell (1898, 1936), who defined it as the interest rate that is consistent with zero inflation. The aforementioned definition of the natural rate follows Woodford (2003).

[2]Again, the natural rate of interest is defined as the real interest rate that prevails or would prevail under flexible prices and wages. Hence, all of the statements about the return to capital carry over to the natural rate of interest in the long run.



Population 16 years and older. Five-year average growth rates (per annum). Numbers in percent. 1950-2015: estimates. 2015-2100: baseline scenario. Source: UN Population Prospects 2015.

Figure 1: Working Age Population Growth - Selected Countries 1950-2100

holds aim at providing the same amount of consumption per person for all generations. Hence, households reduce their saving in response to a fall in the population growth rate, keeping the capital-labor ratio constant. In the long run, the natural rate of interest is independent of the population growth rate. In the Millian case, households maximize the per-person utility of each generation. A fall in population growth does not lead households to reduce their saving rate, given that the size of a generation is not part of their objective function. As in the Solow model, the capital-labor ratio is higher in the long run. This in turn implies a lower steady-state return to capital.

Given the importance of the preference parameter that governs the weight on future generations for the responses to fertility shocks, an estimate of this parameter is needed in order to asses the implications of falling population growth (Figure 1) on the economy and on the natural rate of interest in particular. As pointed out by Mankiw (2005, 317-18):

> "In the end it is clear that the tools of modern growth theory lead to an ambiguous answer about how population growth affects the return to capital. One can write down textbook models in which the two variables move together (the Solow model), and one can write down models in which they do not (the Ramsey model). The natural response to this theoretical ambiguity

> is to muster evidence, either from time-series data or from the international cross section, about the actual effect of population growth."

Following Mankiw's suggestion, I estimate the parameter that governs the relationship between population growth and the natural rate of interest. More specifically, following Becker and Barro (1988), I allow for a more general population weighting function that incorporates the aforementioned preferences as special cases. The model is a standard medium-scale model of the US-economy (Christiano et al., 2003; Smets and Wouters, 2007; Justiniano et al., 2010), that features fertility shocks in addition. I use monthly data on live births in the United States, starting in January 1941, in order to calculate the *natural population growth rate* of the US working age population.[3] This series is then used in the estimation of fertility shocks. I estimate the parameter that governs the curvature of the population weighting function. This parameter equals the steady-state, percentage point change in the natural rate due to a one percentage point permanent change in the population growth rate. The median estimate is 0.67 with 90% probability bands ranging from 0.13 to 1.24. This means that a one percentage point (henceforth pp) permanent decline in the population growth rate leads to a 0.67 pp decline in the natural rate of interest.[4]

The estimated response of the natural rate to fertility shocks has implications for monetary policy. A failure to adjust nominal interest rates appropriately to fertility shocks, leads to persistent changes in the inflation rate. According to my estimates, negative fertility shocks in the US caused a 0.4 pp decline in the natural rate during the 1980s and 1990s. Moreover, inflation rates fell by 0.4 pp over this period due to the delayed response of the central bank to the declining natural rate that has resulted from negative fertility shocks.

Related literature Recent papers studying the economic causes of the US postwar baby boom include Greenwood et al. (2005), Zhao (2014), Doepke et al. (2015), and Jones and Schoonbroodt (2016). By contrast, this paper analyzes the consequences of the postwar baby boom and the subsequent baby bust for the US economy. Jaimovich and Siu (2009) employ panel-data methods to investigate the relationship between the age composition of the labor force and business cycle volatility. They find that demographics account for about 30 percent of the decline in US macroeconomic volatility since the 1980s. This

[3] The natural population growth rate is the growth rate of the population due to the difference between the number of births and the number of deaths.

[4] Conversely, a one pp permanent *increase* in the population growth rate leads to a 0.67 pp *increase* in the natural rate of interest.

paper investigates the effects of fertility shocks, while Jaimovich and Siu (2009) showed how demographics have changed the unconditional moments of US business cycles in recent decades.

Beginning with the Samuelson-Lerner debate, economists have been divided on how to maximize social welfare in the presence of population growth.[5] Classical utilitarianism, following Bentham, calls for maximizing the total sum of individual utility. Average utilitarianism, following Mill, advocates maximizing the utility of the average individual. Depending on the formulation of the welfare function, the interest rate may be equal to the population growth rate, (Samuelson, 1958, 1959), or not, (Lerner, 1959a,b). Arrow and Kurz (1970, 13) reject Samuelson's Millian preference formulation, arguing "that the social felicity is better measured by the sum of all the individual felicity in a given generation; if more people benefit, so much the better." For Rawls (1999, 252-53), by contrast, "maximizing total utility may lead to an excessive rate of accumulation (at least in the near future)."[6] Blanchard and Fisher (1989) employ Millian preferences, while Barro and Sala-i-Martin (1995) advocate a Benthamite formulation of the welfare function. In the context of optimal fertility choice, Becker and Barro (1988) were the first to allow for a more general formulation of preferences.[7] I borrow the general preference specification from Becker and Barro (1988) and estimate the preference parameter using fertility data for the US.

Besides its normative implications, the specification of the intertemporal utility function is particularly important for assessing the consequences of permanent changes in population growth on capital accumulation, as highlighted by Canton and Meijdam (1997). Millian preferences imply a higher capital intensity with lower population growth in the future, as in Yoo (1994), while there is no change in the capital-output ratio with Benthamite preferences, as in Cutler et al. (1990). To the best of my knowledge, this paper is the first to address this theoretical ambiguity empirically using time series data for the US.

[5]For a summary, see Lane (1977) or Nerlove et al. (1987).

[6]"Thus it seems evident, for example, that the classical principle of utility leads in the wrong direction for questions of justice between generations. For if one takes the size of the population as variable, and postulates a high marginal productivity of capital and a very distant time horizon, *maximizing total utility may lead to an excessive rate of accumulation (at least in the near future)* [emphasis added]. Since from a moral point of view there are no grounds for discounting future well-being on the basis of pure time preference, the conclusion is all the more likely that the greater advantages of future generations will be sufficiently large to outweigh most any present sacrifices. This may prove true if only because with more capital and better technology it will be possible to support a sufficiently large population. Thus the utilitarian doctrine may direct us to demand heavy sacrifices of the poorer generations for the sake of greater advantages for later ones that are fare better off" (Rawls, 1999, 252-53).

[7]See also Maußner and Klump (1996) or Baker et al. (2005).

Recently, papers have discussed the threat of a period of "secular stagnation," referring to a term coined by Hansen (1939). Most prominently, Summers (2014) and Eggertsson and Mehrotra (2014) argue that declining, and possibly negative equilibrium real interest rates in combination with a zero lower bound on nominal interest rates may create difficulties in achieving full employment. Lower population growth is considered to be a major trigger of a lower natural rate of interest.[8] In this paper, I estimate the effects of fertility shocks on the natural rate.

Using a New Keynesian overlapping generations model, Carvalho and Ferrero (2014) argue that the deflationary period that Japan has experienced for the last two decades is a result of the central bank's failure to account for the secular decline in the natural rate of interest, resulting from lower population growth and a higher life expectancy.[9] In Carvalho and Ferrero (2014), the relationship between the population growth rate and the natural rate is implied by the life-cycle structure of the model. In this paper, I estimate the relationship between the population growth rate and the natural rate of interest in the US.

The remainder of the paper is organized as follows. Section 2 outlines the model and analyses the link between population growth and the natural rate of interest. Section 3 presents the estimation results and discusses the implications for monetary policy. Section 4 concludes.

2 Model

This section presents a medium-sized dynamic stochastic general equilibrium (DSGE) model with fertility shocks. The feature that distinguishes this paper from the existing business cycle literature (Christiano et al., 2003; Smets and Wouters, 2007; Justiniano et al., 2010) is that it considers a general preference formulation nesting Benthamite and Millian preferences as special cases.[10] The model also features five additional departures from the basic neoclassical growth model: (i) habit formation in consumption, (ii) invest-

[8] "Second, it is well known, going back to Alvin Hansen and way before, that a declining rate of population growth, . . . , means a declining natural rate of interest" (Summers, 2014, 69). "And the equilibrium real interest rate may easily be permanently negative. Forces that work in this direction include a slowdown in population growth, which increases relative supply of savings" (Eggertsson and Mehrotra, 2014, 2).

[9] Kara and von Thadden (2016) calibrate a very similar model for Europe. They project a continued decline of the natural rate of interest in Europe, reflecting the fall in population growth rates and the increase in longevity.

[10] Uhlig (2003) includes stochastic population growth in a Real Business Cycle Model with Millian preferences. Burriel et al. (2010) assume Benthamite preferences in their New Keynesian Model of the Spanish economy.

ment adjustment costs, (iii) variable capital utilization, (iv) monopolistic competition in the goods and labor market, and (v) nominal price and wage rigidities. Habit formation, investment adjustment costs and variable capital utilization are included to capture dynamics following macroeconomic shocks. Monopolistic competition and nominal rigidities are included to account for the effects of monetary policy.

Households There is a continuum of households $j \in [0,1]$, each of size N_t. Each household is a monopolistic supplier of specialized labor, $h_t(j)$. Households trade contingent assets such that in equilibrium consumption and asset holdings are equalized among them (Erceg et al., 2000; Christiano et al., 2005). Households are thus homogeneous only with respect to the amount of labor services they supply and the wage rate that they earn.

The preferences of the household are given by

$$\mathbb{E}_t \left[\sum_{s=0}^{\infty} \beta^s N_{t+s}^{1-\theta} u(c_{t+s}, c_{t+s-1}, h_{t+s}(j)) \right], \tag{1}$$

with $\beta \in (0,1)$. Here, $c_t = C_t/N_t$ is consumption per person and $h_t(j) = H_t(j)/N_t$ are hours worked per person. The instantaneous utility function is compatible with a balanced growth path

$$u(c_t, h_t(j)) = \ln(c_t - \mu c_{t-1}) - v\frac{h_t(j)^{1+\varphi}}{1+\varphi}, \tag{2}$$

with $\mu \in (0,1)$, $\varphi > 0$ and $v > 0$.[11] The parameter μ governs the degree of external habit formation. The size of the household N_t is subject to stochastic shocks ε_t^n and evolves as follows

$$n_t = \ln(N_t/N_{t-1}) = (1-\rho_n)n + \rho_n n_{t-1} + \varepsilon_t^n, \tag{3}$$

with $\varepsilon_t^n \overset{i.i.d.}{\sim} \mathcal{N}(0, \sigma_n^2)$.

It is quite natural to assume that fertility decisions 16 years ago are unaffected by current business cycle conditions.[12] Following Becker and Barro (1988), the parameter

[11] King et al. (1988) prove that these preferences are consistent with a balanced growth path, abstracting from population growth. It turns out that this result carries over to the case with $N_t^{1-\theta}$ appearing in the utility function of the household.

[12] Jaimovich and Siu (2009), who estimate the effect of the age composition of the labor force on the variations in business cycle volatility across G7 countries, make a very similar identification assumption. "Because workforce composition is largely determined by fertility decisions made at least 15 years prior to current volatility, we are able to obtain unbiased inference on the causal effect using standard econometric techniques." (Jaimovich and Siu, 2009, 805)

θ represents the weighting factor with respect to the household size N_t. With $\theta = 0$, the per-capita utility of each generation is weighted by its size (Benthamite preferences). With $\theta = 1$ the per-capita utility of each generation is weighted equally, regardless of its size (Millian preferences).

The flow budget constraint of the household is

$$P_t[C_t + X_t] + B_t \leq \frac{R_{t-1}}{\Xi_t^b} B_{t-1} + (R_t^K u_t - P_t \Psi(u_t))K_{t-1} + W_t(j)H_t(j) + Z_t(j) + D_t. \tag{4}$$

Here, X_t are the purchases of investment goods in period t, B_t are the government bond holdings of the household between t and $t+1$, R_t is the gross nominal return on government bonds between t and $t+1$, P_t is the price of the final good, R_t^K is the rental price of capital, u_t is the capital utilization rate, K_{t-1} is the capital stock chosen by the household in period $t-1$ and rented out to firms in period t, W_t is the nominal wage rate, $Z_t(j)$ is the pay-off from the state contingent claims and D_t is the difference between dividend payments that the household receives from firm ownership in period t and the amount of lump-sum taxes that it pays to the government in period t. Increasing capital utilization is subject to convex adjustment costs $\Psi(u_t)$ with $\Psi'(u_t) > 0$ and $\Psi''(u_t) > 0$. In steady state $\Psi(1) = 0$ and $\psi = \Psi''(1)/\Psi'(1) > 0$.

Ξ_t^b is a risk-premium shock that drives a wedge between the risk-free rate and the risky return on capital (Smets and Wouters, 2007). It follows the shock process

$$\zeta_t^b = \ln(\Xi_t^b) = \rho_b \zeta_{t-1}^b + \varepsilon_t^b, \tag{5}$$

with $\rho_b \in (0, 1)$ and $\varepsilon_t^b \overset{i.i.d.}{\sim} \mathcal{N}(0, \sigma_b^2)$.

The budget constraint expressed in per-capita terms is

$$P_t[c_t + x_t] + b_t \leq \frac{R_{t-1}}{\Xi_t^b}\frac{N_{t-1}}{N_t} b_{t-1} + (R_t^K u_t - P_t \Psi(u_t))\frac{N_{t-1}}{N_t} k_{t-1} + W_t(j)h_t(j) + z_t(j) + d_t, \tag{6}$$

where small letters denote per-capita quantities.

The capital stock per person evolves according to

$$k_t \leq (1-\delta)\frac{N_{t-1}}{N_t} k_{t-1} + \Xi_t^x (1 - S(x_t/x_{t-1}))x_t, \tag{7}$$

with $\delta \in (0, 1)$, and where $S(x_t/x_{t-1})$ captures convex adjustment costs to investment. In steady state $S(1) = S'(1) = 0$ and $\varsigma = S''(1) > 0$. The functional forms of Ψ and S are the same as in Christiano et al. (2005). Ξ_t^x is an investment shock that affects the efficiency with which the final good can be transformed into physical capital (Justiniano et al., 2010). It follows the shock process

$$\xi_t^x = \ln(\Xi_t^x) = \rho_x \xi_{t-1}^x + \varepsilon_t^x, \tag{8}$$

with $\rho_x \in (0,1)$ and $\varepsilon_t^x \overset{i.i.d.}{\sim} \mathcal{N}(0, \sigma_x^2)$.

Employment agencies Perfectly competitive employment agencies combine the different amounts of specialized labor $H_t(j) = N_t h_t(j)$ into a homogenous labor input that they sell to intermediate good firms

$$H_t = \left(\int_0^1 H_t(j)^{\frac{\epsilon_{w,t}}{\epsilon_{w,t}-1}} dj \right)^{\frac{\epsilon_{w,t}-1}{\epsilon_{w,t}}}, \quad \epsilon_{w,t} = \epsilon_w \Xi_t^w, \tag{9}$$

where $\epsilon_w > 1$ is the elasticity of substitution between the different labor services. Ξ_t^w is a wage markup shock that affects the desired markup of wages over the marginal rate of substitution between consumption and leisure of households. The wage markup shock can be equally interpreted as labor supply shock (Smets and Wouters, 2003). It follows the shock process

$$\xi_t^w = \ln(\Xi_t^w) = \rho_w \xi_{t-1}^w + \varepsilon_t^w - \vartheta_w \varepsilon_{t-1}^w, \tag{10}$$

with $\rho_w \in (0,1)$, $\vartheta_w \in (0,1)$ and $\varepsilon_t^w \overset{i.i.d.}{\sim} \mathcal{N}(0, \sigma_w^2)$. The optimal demand for specialized labor j is given by

$$H_t(j) = \left(\frac{W_t(j)}{W_t} \right)^{-\epsilon_{w,t}} H_t. \tag{11}$$

Wage setting Every period, a random fraction $\lambda_w \in (0,1)$ of households cannot reset their nominal wage. For them wages evolve according to the indexation rule

$$W_t(j) = W_{t-1}(j) \pi_{t-1}^{\iota_w} \pi^{1-\iota_w}, \tag{12}$$

where $\pi_t = \ln(P_t / P_{t-1})$ and π is the steady-state inflation rate.

The remaining fraction sets their wage in order to maximize

$$\mathbb{E}_t \left[\sum_{s=0}^{\infty} \beta^s N_{t+s}^{1-\theta} \left(-v \frac{h_{t+s}(j)^{1+\varphi}}{1+\varphi} + Q_{t,t+s} W_t(j) \pi^{(1-\iota_w)s} \prod_{k=1}^{s} \pi_{t+k-1}^{\iota_w} h_{t+s}(j) \right) \right], \tag{13}$$

subject to (11).

Final good firms There is a perfectly competitive final good sector. The final consumption good is produced using the following technology

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{\epsilon_{p,t}-1}{\epsilon_{p,t}} \Xi_t^p} di \right)^{\frac{\epsilon_{p,t}}{\epsilon_{p,t}-1} \Xi_t^p}, \quad \epsilon_{p,t} = \epsilon_p \Xi_t^p, \tag{14}$$

where $Y_t(i)$ denotes the quantity of intermediate good i that is used in the production of the final good, and where $\epsilon_p > 1$ is the elasticity of substitution between the different intermediate goods. Ξ_t^p is a price markup shock, that affects the desired markup of price over marginal costs. It follows the shock process

$$\zeta_t^p = \ln(\Xi_t^p) = \rho_p \zeta_{t-1}^p + \varepsilon_t^p - \vartheta_p \varepsilon_{t-1}^p, \tag{15}$$

with $\rho_p \in (0,1)$, $\vartheta_p \in (0,1)$ and $\varepsilon_t^p \overset{i.i.d.}{\sim} \mathcal{N}(0, \sigma_p^2)$. The optimal demand for good i is given by

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon_p} Y_t. \tag{16}$$

Intermediate-goods firms There is a continuum of intermediate-goods firms indexed by $i \in [0,1]$. Firm i is the monopoly supplier of good i. All firms use the same technology, represented by the production function

$$Y_t(i) = K_t(i)^{\alpha} H_t(i)^{1-\alpha}, \tag{17}$$

with $\alpha \in (0,1)$, and where $K_t(i)$ and $H_t(i)$ are the capital and labor services demanded by firm i. Factor markets are perfectly competitive. This together with the constant-returns-to-scale technology (17) ensures that marginal costs are identical across firms. Each period, a random fraction $\lambda_p \in (0,1)$ of firms cannot adjust their price. For them prices evolve according to the indexation rule

$$P_t(i) = P_{t-1}(i) \pi_{t-1}^{\iota_p} \pi^{1-\iota_p}, \tag{18}$$

with $\iota_p \in (0,1)$.

A firm that can adjust its price in period t maximizes

$$\mathbb{E}_t \left[\sum_{s=0}^{\infty} \lambda_p^s \beta^s N_{t+s}^{1-\theta} Q_{t,t+s} (P_t(i) \pi^{(1-\iota_p)s} (\prod_{k=1}^{s} \pi_{t+k-1}^{\iota_p}) - MC_{t+s}) Y_{t+s}(i) \right], \tag{19}$$

subject to (16). Here, $Q_{t,t+s}$ is the stochastic discount factor for nominal profits in period $t+s$, and where MC_{t+s} denote nominal marginal costs in period $t+s$.

Monetary authority The monetary authority sets the nominal interest rate according to the following rule

$$R_t = R_{t-1}^{\rho_r} \left[R \Pi_t^{\phi_\pi} (Y_t / Y_t^n)^{\phi_y} \right]^{1-\rho_r} e^{\varepsilon_t^r}, \tag{20}$$

with $\rho_r \in (0,1)$, $\phi_\pi > 1$, $\phi_y \geq 0$ and where $\varepsilon_t^r \overset{i.i.d.}{\sim} \mathcal{N}(0, \sigma_r^2)$ is a monetary policy shock. Here, R is the steady-state nominal interest rate, $\Pi_t = P_t / P_{t-1}$ is the gross inflation rate, and Y_t^n the natural level of output, i.e. the level of output that would prevail under flexible prices and wages, and no markup shocks (Woodford, 2003).

Fiscal authority The government purchases $G_t = gY\Xi_t^g$ amount of the final good following, which is subject to government spending shocks

$$\xi_t^g = \ln(\Xi_t^g) = \rho_g \xi_{t-1}^g + \varepsilon_t^g, \tag{21}$$

with $\rho_g \in (0,1)$ and $\varepsilon_t^g \overset{i.i.d.}{\sim} \mathcal{N}(0, \sigma_g^2)$. Here, g is the steady-state government share of GDP. The fiscal authority finances its government expenditure either through lump-sum taxes paid by households or by issuing one-period government bonds. Monetary policy is active, while fiscal policy is passive in the sense of Leeper (1991).

Market clearing and equilibrium The market clearing conditions for capital and labor services are

$$\int_0^1 K_t(i)di = K_{t-1}u_t, \tag{22}$$

$$\int_0^1 H_t(i)di = H_t. \tag{23}$$

The aggregate resource constraint is

$$Y_t = C_t + X_t + G_t + \Psi(u_t)K_{t-1}. \tag{24}$$

The relationship between population growth and the natural rate of interest Canton and Meijdam (1997) show within a neoclassical growth model how the responses to a change in the population growth rate depend on the preference specification of households. A fertility shock has a threefold impact on the economy in this model. First, a higher population growth rate means that, at least temporarily, less capital is available per worker. This reduces output per person in the short run. At the same time, factor prices are affected. Wages fall together with the capital-labor ratio, whereas the rental price of capital increases. Second, a higher population growth rate implies a faster depreciation of the per-person capital stock, since new members of the household are born without any capital. A fertility shock thus resembles a depreciation shock, as studied by Ambler and Paquet (1994), and also a capital-quality shock, as studied by Gertler and Kiyotaki (2010).[13] Third, fertility shocks may affect the intertemporal optimality condition of the household. Ignoring ζ_t^b and setting $\mu = 0$, the linearized intertemporal optimality condition of the household is then

$$\mathbb{E}_t[\widehat{c}_{t+1}] - \widehat{c}_t = (\widehat{R}_t - \mathbb{E}_t[\widehat{\pi}_{t+1}]) - \theta \mathbb{E}_t[\zeta_{t+1}^n]), \tag{25}$$

where $\widehat{c}_t = \ln(c_t/c)$, $\widehat{R}_t = \ln(R_t/R)$, $\widehat{\pi}_t = \ln(\pi_t/\pi)$ are log-deviations of c_t, R_t and π_t from their respective steady-state levels and $\zeta_t^n = n_t - n$.

With $\theta = 0$, the household is willing to provide the additional resources for investment needed to keep the level of capital per person costant in the long run. This is because the household weights each generation by its size and because larger future generations need a larger total capital stock to attain the same level of consumption and leisure as the present generation. With $\theta > 0$, on the other hand, the generation weights $N_t^{1-\theta}$ increase by less than one-to-one with the population size. The household is unwilling to sustain the same level of capital per person. As a consequence, expected per-capita consumption growth is negative, i.e. $\mathbb{E}_t[\widehat{c}_{t+1}] - \widehat{c}_t \downarrow$, or/and the real interest rate increases, i.e. $\widehat{R}_t - \mathbb{E}_t[\widehat{\pi}_{t+1}] \uparrow$.

Another insight from (25) is that θ corresponds to the long-run, percentage point change of the natural rate of interest, given a permanent one percentage point increase in the population growth rate. Suppose that in period 0 the population growth rate increases permanently to a higher level $n_t = n^* > n$. Further, suppose that the economy has converged to the new steady state by period t, i.e. $\widehat{c}_t = \mathbb{E}_t[\widehat{c}_{t+1}]$. Thus, the natural rate of interest, in log-deviations from its old steady-state level, is

$$\widehat{R}_t - \mathbb{E}_t[\widehat{\pi}_{t+1}] = \theta \zeta^n, \tag{26}$$

with $\zeta^n = n^* - n > 0$. Whether the real interest rate moves with population growth in

[13] See also Liu et al. (2011); Furlanetto and Seneca (2014). A crucial difference is that a higher population growth rate not only increases the depreciation or depletion of the capital stock (7), but also of others assets such as bonds (6).

the long run thus crucially depends on the preference parameter θ.

3 Estimation

Population growth in the United States Using monthly data on live births from the *National Center for Health Statistics (NCHS)*, I construct a quarterly time series for the "natural" growth rate of the US working age population. The natural growth rate is the percentage change of the working age population that is due to the difference between past births and current deaths. It is calculated as follows

$$n_t = \frac{b_{t-16y,t} \times Births_{t-16y} - Deaths_t}{N_t}, \tag{27}$$

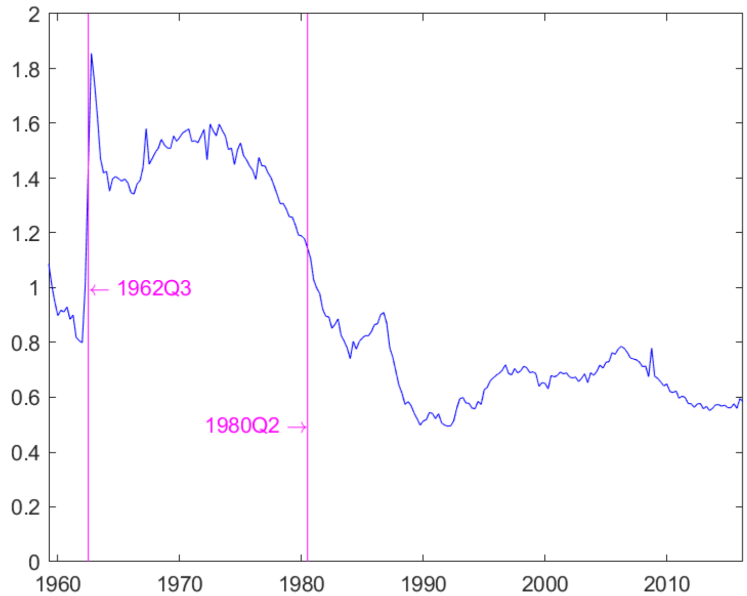
where ν_t is the natural population growth rate, $b_{t-16y,t}$ is the fraction of persons surviving to age 16, $Births_{t-16y}$ is the number of live births 16 years ago, $Deaths_t$ is the total number of deaths, 16 years and older, and N_t is the civilian noninstitutional population 16 years and older.[14] Appendix A contains further details on the data. Figure 2 presents the natural growth rate of the US working age population between 1957Q1 and 2016Q2.[15] The baby boomer cohorts joined the working age population between 1962Q3 and 1980Q2, creating an increase in the growth rate from about one pp per year in the early 1960s to an average rate of about 1.5 pp during the 1960s and 1970s.[16] The following baby bust of the 1980s led to a one pp decline in the growth rate. The natural population growth rate slightly increased again in the late 1990s and the early 2000s, due to the arrival of the so called "echo boomer," i.e. the children of the baby boomer, to the working age population. The current natural growth rate of the US population 16 years and older is about 0.6 pp.

Calibrated parameters The discount factor is set such as to match an annual real interest rate of 4%. The capital deprecation rate is set such as to match a quarterly investment-to-capital ratio of 0.025. The capital income share is $\alpha = 0.36$. The elasticity of substitution between intermediate goods is $\epsilon_p = 10$. The elasticity of substitution between the different types of labor is $\epsilon_w = 10$. The government consumption share of GDP is $g = 0.2$.

[14]The natural population growth rate is the population growth rate due to (past) fertility and current mortality rates. The other component of population growth is net migration.

[15]The monthly time series for live births in the United States starts in January 1941. This cohort entered the civilian noninstitutional population 16 years and older in the first quarter of 1957.

[16]The baby boomer cohorts are the cohorts born between mid-1946 and mid-1964.



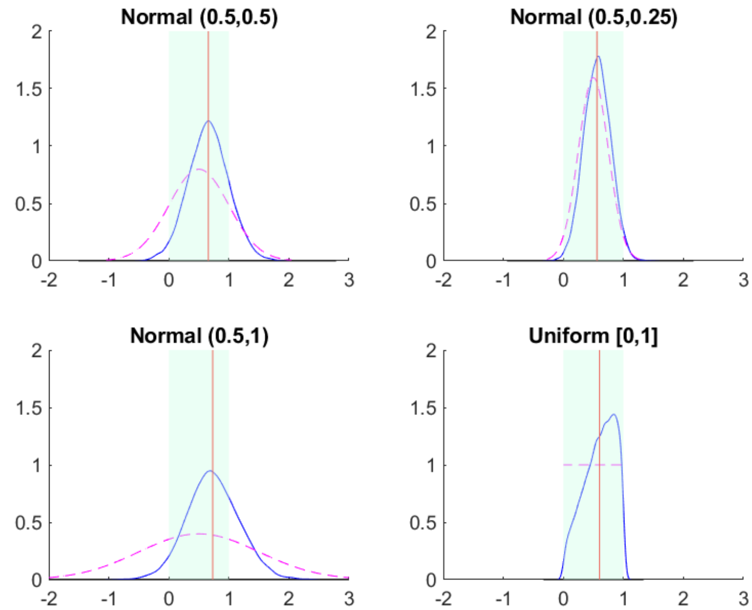
Contribution of fertility shocks to the growth rate of the civilian noninstitutional population, 16 years and older. Annualized percentage points.

Figure 2: US Fertility Shocks 1957Q1-2014Q4

Estimated parameters and shocks processes The model is estimated with Dynare employing Bayesian methods. Apart from population growth, the same macroeconomic time series as in Justiniano et al. (2010) is used. The sample ranges from 1959Q2 to 2016Q2. See appendix A for details on the population data.

The estimates of parameters and shock processes can be found in appendix B. The average natural population growth rate is $\nu = 0.22/100$. Fertility shocks are highly persistent. The persistence parameter is $\rho_n = 0.979$. The standard deviation of fertility shocks is $\sigma_n = 0.015/100$, implying that a one standard deviation positive fertility shock corresponds to a 0.06 pp increase in the annualized population growth rate.

The baseline estimate of θ is 0.66 with the 90th-percent interval ranging from 0.13 to 1.24. Given this prior, the case of Benthamite preferences can be ruled out, whereas the case of Millian preferences cannot. The data proves to be informative about θ, shifting the mean estimate to the right of the prior mean of 0.50. In order to asses the sensitivity of results, the model is re-estimated with three other prior specifications (Figure 3). In the first and second case, the standard deviation of the normally-distributed prior is set to 0.25 and 1, respectively. With a the relatively tight prior, the posterior distribution of θ closely follows the prior. The likelihood is not informative enough compared to the prior. The mean estimate of θ is at 0.56, closer to the prior mean. With a wider



Solid lines: posterior distribution. Dashed lines: prior distributions.

Figure 3: Prior and Posterior Distributions of θ

prior, the posterior mean of θ is 0.73. At the same time, the range of estimates is larger, reflecting the increased prior-uncertainty about θ. In the fourth case, a uniform prior over the interval $[0, 1]$ is considered. The mean estimate is 0.60, close the baseline case. In summary, these estimates point to a positive value of θ, and thus to a positive long-run relationship between the population growth rate and the natural rate of interest.[17]

Impulse responses Figure 4 plots the impulse responses to a one standard deviation positive fertility shock at the estimated parameter values. Per-capita output and consumtion fall, reflecting the decline in the capital stock per worker due to the increased population growth rate. Investment increases when $\theta = 0$ (dashed line), but falls when $\theta = 1$ (dash-dotted line). Consequently, the capital stock per person is persistently lower in the latter case, whereas the increase in investment dampens the decline of the per-person capital stock in the first case. With a higher expected population growth rate

[17]In an earlier version of this paper, I estimated θ by matching the model-based impulse responses to fertility shocks with empirical responses form a structural vector autoregressive (VAR) model. First, I included the natural population growth rate ν_t in a VAR model of the US economy. I employed a recursive identification in which the natural population growth rate is ordered first. One-step-ahead forecast errors in ν_t are due to fertility shocks only. It is quite natural to assume that fertility decisions 16 years ago are unaffected by current business cycle conditions.[18] Second, I estimated θ for different model calibrations using a minimum distance estimator (Christiano et al., 2005). The parameter estimates ranged from 0.3 to 0.6.

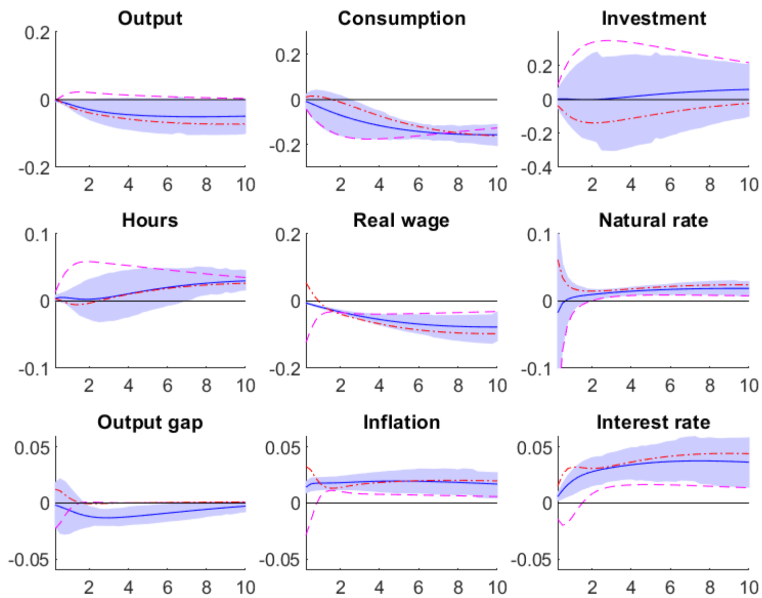
in the future, more resources are needed for investment, in order to sustain the current level of capital per person. Even absent capital depreciation, the capital stock per person falls, unless new capital is accumulated in the meantime. Hours worked unambiguously increase after five years reflecting the fall in consumption, which offsets the negative substitution effect from lower wages. With Benthamite preferences, hours worked increase on impact already, inducing a marked decline in the real wage. The increase in hours worked and investment help stabilize output per person and mitigate the decline in consumption per-capita that comes along with higher investment.

Depending on the preference specification, the short-run response of the natural rate is positive ($\theta = 1$) or negative ($\theta = 0$). After about two years the estimated response of the natural rate is unambigously positive, though. The output gap falls slightly after about two years. A fertility shock has two opposing effects on the output gap. First, a higher population growth leads to an increase in the natural rate and in inflation. The central bank reacts by increasing the short-term nominal interest rate. Due to its delayed response, however, real interest rates increase by less than the natural rate. Therefore, output is higher than in the case of flexible prices.[19] The second effect comes from sticky wages. A higher population growth rate induces a downward pressure on real wages. This holds under all preference specifications. Since nominal wages are sticky, real wages fall not as much as in the case with flexible nominal wages, implying a negative output gap. According to the estimated responses the latter effect dominates.

Most importantly, inflation increases persistently in response to a positive fertility shock, while the FFR increases with a lag only. The higher inflation rate reflects above all the rising natural rate of interest that is not entirerly offset by the central bank.

As noted before, the parameter θ represents the long-run percentage change in the natural rate of interest due to a one percentage change in the population growth rate. How do the estimates from this paper relate to the long-run relationship between the population growth rate and the natural rate of interest that is implied by other growth models? Table 1 compares the steady-state relationship between the population growth rate and the natural rate of interest across different models: the neoclassical growth model of Solow (1956), the overlapping generations model of Weil (1989), the overlapping generations model of Auerbach and Kotlikoff (1987), and the overlapping generations model of Gertler (1999). All models predict a positive link between population growth and the natural rate. The Solow model implies the largest impact of population growth on the natural rate. The reason is that the gross saving rate is constant in the Solow model. An increase in the population growth rate leads to a fall in the capital-

[19] Setting the $\lambda_w = 0$ (degree of wage stickiness) results in a slightly positive response of the output gap.



Solid lines: median. Shaded areas: 90% probability bands. Dashed lines: Benthamite preferences ($\theta = 0$). Dash-dotted lines: Millian preferences ($\theta = 1$). Years on x-axis. Percentage point deviations from steady state.

Figure 4: Estimated Impulse Responses to a Fertility Shock

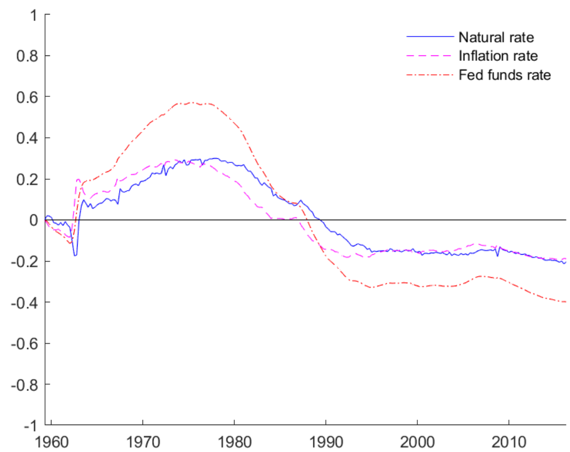
labor ratio, and consequently to a large increase in the natural rate. The other models suggest a smaller, but still sizable long-run reaction of the natural rate to permanent changes in population growth. What these models have in common, is that agents do not fully internalize the dilution of the capital stock through a higher population growth rate. This is either because saving is exogenous as in the Solow model, or because of overlapping generations without intergenerational bequest motives. Only in the representative agent model with Benthamite preferences, the response of households is *exactly* such that the long-run natural rate of interest does not respond to a change in the population growth rate. The results of this paper support the predictions of the other models, namely that there is a positive link between population growth and the natural rate of interest.

Historical decomposition Recent decades have seen not only a fall in population growth, but also a substantial decline in inflation and interest rates. Today, both inflation and interest rates are near zero. Given the estimated θ, roughly 0.4 pp of the decline in inflation can be attributed to falling population growth (Figure 5). In response to the deflationary pressure steming from falling population growth, the federal funds rate was lowered by

Table 1: Population Growth and the Natural Rate - Model Comparison

	Δr
Solow (1956)	1.27
Weil (1989)	0.28
Auerbach and Kotlikoff (1987)	0.26
Gertler (1999)	0.70

Steady-state comparative statics: $\Delta r = r(n = 1\%) - r(n = 0\%)$. Here, r is the steady-state real interest rate, and n is the steady-state population growth rate.



Years on x-axis. Numbers in percentage points.

Figure 5: Historical Decomposition of Interest Rates and Inflation

about 1 pp over the course of the last decades. [20] To summarize, changes in the US population growth rate are a nonnegligible, but not a major, driver of the secular decline in inflation and interest rates.

4 Conclusion

In this paper, I estimate the macroeconomic responses to fertility shocks in a medium-scale New Keynesian model. The responses depend on the weighting of the different generations in the utility function of households. Of particular interest is the response of the natural rate of interest. When generations are weighted by their size (Benthamite preferences), there is no link between population growth and the natural rate in the long run. When the utility of different generations is maximized irrespective of their size (Millian preferences), there is a one-to-one link between population growth and the

[20] This does not mean that the FED directly responded to lower population growth, but that it lowered nominal interest rates in response to falling inflation rates.

long-run natural rate.

The mean estimate of the parameter that governs the weighting of different generations in the utility function of households is about 0.67, implying that a one percentage point decrease in the steady-state population growth rate leads to 0.67 decrease in the steady-state natural rate of interest. Lower population growth in the aftermath of the US baby-boom-and-bust cycle has lead to a reduction in the natural rate of interest of about 0.4 percentage points. According to these estimates, negative fertility shocks lowered inflation by about 0.4 pp during the 1980s and early 1990s.

In summary, this paper confirms the existence of a natural rate channel, through which lower population growth exerts downward pressure on inflation and interest rates. The magnitude is moderate, however, at least compared to the simulation results of Carvalho and Ferrero (2014) for Japan. This might be due to the more pronounced demographic transition in Japan compared to the United States (Figure 1). Both Carvalho and Ferrero (2014) and this paper consider closed economies. An open-economy extension of the model may help quantifying the role of such cross-country differences in population growth for interest rates and the conduct of monetary policy.

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A Population Data

Table 2: Population Data

VARIABLE	FREQ.	CODE	SOURCE
Civilian noninstitutional population, 16+	monthly	LNU00000000	BLS/CPS
Number of persons surviving to age 16	decennial		NCHS
Total number of live births	monthly		NCHS
Total number of deaths, 15+[1]	annual		NCHS

$Births_t$ is seasonally adjusted using X-13 ARIMA-SEATS quarterly seasonal adjustment method. The numbers for $b_{t-16y,t}$ and $Deaths_t$ are interpolated to quarterly frequency. This is of course only an approximation. Given the absence of major epidemics, wars, etc. in recent decades, both series are probably very smooth at a quarterly frequency, though.

B Estimation results

Table 3: Estimates (parameters)

	COEFFICIENT	PRIOR			POSTERIOR		
		DENSITY	MEAN	STD.	MEAN	5%	95%
θ	Population weight	$\mathcal{N}$	0.5	0.25	0.66	0.13	1.24
μ	Consumption habit	$\mathcal{B}$	0.4	0.15	0.74	0.70	0.79
ψ	Variable capital utilization	$\mathcal{G}$	2	0.5	4.17	3.16	5.09
ς	Investment adjustment cost	$\mathcal{G}$	4	1	2.58	1.45	3.65
φ	Frisch elasticity (inverse)	$\mathcal{G}$	2	0.5	2.52	1.62	3.31
λ_p	Price stickiness	$\mathcal{B}$	0.5	0.15	0.81	0.77	0.85
λ_w	Wage stickiness	$\mathcal{B}$	0.5	0.15	0.67	0.55	0.77
ι_p	Inflation indexation	$\mathcal{B}$	0.2	0.1	0.21	0.08	0.33
ι_w	Wage indexation	$\mathcal{B}$	0.2	0.1	0.55	0.36	0.74
ρ_r	Interest smoothing	$\mathcal{B}$	0.7	0.15	0.81	0.78	0.84
ϕ_π	Inflation coefficient	$\mathcal{G}$	2	0.2	2.19	1.96	2.41
ϕ_y	Output gap coefficient	$\mathcal{G}$	0.15	0.05	0.12	0.09	0.14

$\mathcal{N}$: Normal distribution. $\mathcal{G}$: Gamma distribution. $\mathcal{B}$: Beta distribution. $\mathcal{IG}$: Inverse Gamma distribution. Log data density is -1540.17. 125,000 replications. Burn-in: 62,500. 2 chains. Acceptance ratios: 28.02% and 28.22% for chain 1 and 2.

[1] Only data for the population 15+ is available.

Table 4: Estimates (shock processes)

	Coefficient	Prior			Posterior		
		Density	Mean	Std.	Mean	5%	95%
100ν	Population growth (st. st.)	$\mathcal{N}$	0.4	0.15	0.23	0.16	0.30
100γ	GDP growth (st. st.)	$\mathcal{N}$	0.4	0.15	0.33	0.29	0.36
400π	Inflation (st. st.)	$\mathcal{N}$	2	0.5	1.83	1.36	2.34
$400r$	Fed funds rate (st. st.)	$\mathcal{N}$	2	0.5	2.49	1.79	3.17
ρ_n	Fertility AR(1)	$\mathcal{B}$	0.8	0.1	0.98	0.97	0.99
ρ_a	Technology AR(1)	$\mathcal{B}$	0.8	0.1	0.97	0.96	0.98
ρ_b	Risk premium AR(1)	$\mathcal{B}$	0.8	0.1	0.85	0.80	0.89
ρ_x	Investment AR(1)	$\mathcal{B}$	0.8	0.1	0.94	0.91	0.98
ρ_g	Government AR(1)	$\mathcal{B}$	0.8	0.1	0.95	0.93	0.97
ρ_p	Price markup AR(1)	$\mathcal{B}$	0.8	0.1	0.97	0.95	0.99
ρ_w	Wage markup AR(1)	$\mathcal{B}$	0.8	0.1	0.84	0.69	0.98
ϑ_p	Price markup MA(1)	$\mathcal{B}$	0.5	0.15	0.74	0.64	0.84
ϑ_w	Wage markup MA(1)	$\mathcal{B}$	0.5	0.15	0.82	0.67	0.96
σ_n	Fertility (std., in %)	$\mathcal{IG}$	0.1	1	0.01	0.01	0.02
σ_a	Technology (std., in %)	$\mathcal{IG}$	0.5	1	0.62	0.57	0.66
σ_b	Risk premium (std., in %)	$\mathcal{IG}$	0.5	1	0.45	0.33	0.58
σ_x	Investment (std., in %)	$\mathcal{IG}$	0.5	1	4.95	3.85	6.07
σ_g	Government (std., in %)	$\mathcal{IG}$	0.5	1	0.42	0.38	0.45
σ_p	Wage markup (std., in %)	$\mathcal{IG}$	0.1	1	0.16	0.12	0.19
σ_w	Wage markup (std., in %)	$\mathcal{IG}$	0.1	1	0.73	0.66	0.80
σ_r	Monetary policy (std., in %)	$\mathcal{IG}$	0.1	1	0.24	0.22	0.26

$\mathcal{N}$: Normal distribution. $\mathcal{G}$: Gamma distribution. $\mathcal{B}$: Beta distribution. $\mathcal{IG}$: Inverse Gamma distribution. Log data density is -1540.17. 125,000 replications. Burn-in: 62,500. 2 chains. Acceptance ratios: 28.02% and 28.22% for chain 1 and 2.