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Abstract

This paper addresses the importance of compositional changes in the labor force for the development of the wage distribution. Demographic change and higher educational attainment imply a shift toward employees with more experience and/or better education. These groups are characterized by higher relative wages as well as higher within-group wage inequality. Mechanically, these compositional shifts entail a rise in wage inequality. We demonstrate this mechanism theoretically and present empirical evidence using data of the German Socio-Economic Panel from the mid 1990's to 2012. Accounting for the parallel changes in the age structure and the educational background of the labor force, the compositional effects alone can explain up to one quarter of the observed increase in aggregate wage inequality.

Keywords: wage structure, inequality decomposition, demographic change, population aging, education, skills

JEL classifications: J31; D31; J11

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1 Introduction

The development of the income distribution has become an integral part of economic and social policy debates worldwide. The core of these debates revolves around the observation that income inequality has been rising over the past decades as well as its possible causes and consequences (see, e.g., OECD 2008, 2011). Germany is no exception in this regard. Income inequality, in particular wage inequality, has been trending upward since reunification, which led to calls for more income re-distribution measures to be taken by the government during the 2013 election campaigns and the imminent introduction of a federal minimum wage in 2015.

The public debate about income inequality often takes place at the aggregated level although the reasons behind these developments are manifold. For instance, the factors that are discussed as influencing the distribution of household income in Germany include – among others – the contribution of different income sources (e.g., capital income, Fräßdorf et al. 2011, Rehm et al. 2014), changing household structures and assortative mating (see, e.g., Peichl et al. 2012, Pestel 2014), or the tax and transfer system (e.g., Bartels and Bönke 2013, Fuest et al. 2008). Our paper contributes to the economic literature on income inequality by highlighting the importance of compositional changes due to demographic and educational developments, in particular with regard to the wage distribution.

The distribution of personal labor income (i.e., the wage structure) is of particular interest because labor earnings represent the most important income source for most households. Like the distribution of household income, the German wage distribution has become more dispersed over the past two decades. Wage inequality started to increase in the late 1980's in the upper half of the wage distribution and, beginning in the 1990's, it also started to rise in the lower part (Dustmann et al. 2009). Biewen and Juhasz (2012) and Schmid and Stein (2013) attribute significant parts of the increase in total household income inequality in Germany to a rise in the dispersion of labor earnings, especially during the first half of the 2000's. The reasons for the rise in labor income inequality, especially wage inequality, are subject to intense discussions. They include technological change, increased global competition and institutional factors such as minimum wages or de-unionization (for recent surveys, see GCEE 2012, Fitzenberger 2012, Dustmann et al. 2014).

While the contribution of different factors for the determination of individual wages and thereby wage inequality has been studied quite intensively, less attention has been devoted to the compositional changes within the labor force that are taking place constantly (exceptions are Dustmann et al. 2009, Gernandt and Pfeiffer 2007). This is insofar surprising as structural shifts in the labor force toward groups with relatively high wage inequality, e.g., older or more educated employees, mechanically induce an increase in aggregate inequality measures. Demographic change in the form of population aging as well as higher educational attainment are major trends shaping the current and future economic and societal development, especially in Germany.

Most of the discussion concerning demographic changes center around its potential effects on labor supply, the productivity of older employees and the financial burden on the social security system (see, e.g., GCEE 2011, Börsch-Supan 2014, Fertig and Schmidt 2003). In contrast, the influence of an aging labor force on earnings inequality has been much less discussed in Germany so far. Lemieux (2006) argues that a large fraction of the increase in wage inequality in the US between 1973 and 2003 can be explained by composition effects with regard to experience and education (a view challenged by Autor et al. 2008). Mookherjee and Shorrocks (1982) attribute a large share of the rise in household income inequality in the UK between 1965 and 1980 to changes in the age structure of the population. Dustmann et al. (2009) find that compositional changes account for a significant part of the increase in earnings inequality in West Germany between 1975 and 2004, particularly in the upper half of the income distribution. Antonczyk et al. (2010) report that rising skill premia between educational groups can be explained partly by age and cohort effects which highlights that age and skill composition change in parallel over time and should be disentangled. Looking at West and East Germany, Gernandt and Pfeiffer (2007) attribute only a small influence to composition effects, mainly among workers with low tenure and in East Germany.

Departing from the idea that demographic change paired with increasing educational attainment (motivated by the return on educational investment) causes a structural shift in the composition of the labor force toward groups with relatively high and more dispersed individual wages, we show how such a development mechanically causes earnings inequality to rise. Using data of the German Socio-Economic Panel, we empirically illustrate the theoretical predictions: Approximately 10% of the overall increase in wage inequality from the mid 1990's to the 2010's can be attributed to plain composition effects in the age structure. In combination with shifts in the educational backgrounds, compositional changes can account for up to one quarter of the inequality increase. While a rise in unexplained residual inequality remains, in particular regarding the short time period from the end of the 1990's to the mid 2000's which was marked by a strong deterioration of the labor market, the analysis underpins the significant role of structural shifts for an aggregate economic outcome such as the wage distribution.

The remainder of this paper is organized as follows: In order to derive a prediction about how wage inequality is altered by population aging, we present a stylized theoretical model in Section 2 and apply the model to a specific measure of inequality in Section 3. Section 4 presents the data and our empirical approach. The descriptive results are discussed in Section 5. Section 6 concludes.

2 Theoretical background

In order to study how the age and skill distribution within the labor force affects wage inequality, we develop a stylized model of heterogeneous workers that features an aggregate wage distribution which is the mixed distribution of two different cohorts. The model economy is a small-open economy, i.e., all goods prices are exogenously set at world markets. [1] Workers are heterogeneous with respect to their abilities or skills a.[2] Abilities are continuously distributed over the support $[0, 1]$ according to a general density function $f(a)$. The total population is normalized to mass one and the shares of old and young workers are denoted by π and $1 - \pi$ respectively. The age structure is assumed exogenous as we do not explicitly model fertility.

Without any further education, an individual supplies $1 + a$ efficiency units of low-skilled labor when young. When old, a low-skilled worker supplies $(1 + a)\,\varepsilon_L$ efficiency units with $\varepsilon_L > 1$ denoting productivity growth due to experience gained in the first period. Alternatively, a young individual can spend an exogenously given fraction of time λ to enter the education system and supply a total of $(1 + ba)(1 - \lambda)$ efficiency units of high-skilled labor when young.[3]

[1]The model is an extension of the theoretical framework of Meckl and Zink (2004) and Meckl and Weigert (2003)

[2]We interpret ability as a mixture of innate ability and knowledge acquired during compulsory schooling. The educational choice is modeled as a choice of further education.

[3]We consider the time to educate as an institutional parameter set by the curriculum of the respective

The parameter $b > 1$ measures the gross effect of education on marginal efficiency units of a trained worker with ability a. In case of an educational investment, only the fraction $(1 - \lambda)$ of efficiency units can be used for work in the first period. When old, a trained high-skilled individual supplies a total of $(1 + ba)\varepsilon_H$ efficiency units with $\varepsilon_H > 1$ denoting productivity growth due to experience gained. As high-skilled workers devote some of their time in period one to learning, they have less time to gain experience while working. Therefore, it may well be the case that $\varepsilon_H < \varepsilon_L$. We assume that in the production process both types of labor are *qualitatively* different, i.e., a low-skilled worker cannot work as a high-skilled worker, and vice versa. Accordingly, low-skilled workers earn a wage rate of w_L per efficiency unit while high-skilled earn w_H per efficiency unit.

To simplify the model, individual preferences are defined over the consumption of a homogeneous good Y and assumed identical for all workers. Saving is allowed with the interest rate set to zero. Therefore, an agent maximizes his total lifetime wage income by choosing to educate if the ability a is higher than a threshold α, defined by:

$$\alpha = \{a : (1 + a)(w_L + w_L \varepsilon_L) = (1 + ba)(w_H (1 - \lambda) + w_H \varepsilon_H)\} \tag{1}$$

Workers with ability α are indifferent between investing in education or entering the labor market immediately to work as low-skilled. The threshold value of each cohort depends on the exogenous relative factor price $\omega \equiv \frac{w_H}{w_L}$ and on the exogenous parameters b and λ.The threshold is given by the intersection of the two lifetime wage functions and can be calculated as:

$$\alpha = -\frac{\omega((1 - \lambda) + \varepsilon_H) - (1 + \varepsilon_L)}{b\omega((1 - \lambda) + \varepsilon_H) - (1 + \varepsilon_L)}. \tag{2}$$

The parameters b, λ, ε_L, ε_H and the relative wage ω have to satisfy the two boundary conditions such that $\alpha \in (0, 1)$. To this end, we assume that an individual with ability $a = 0$ should always choose to work as a low-skilled in order to maximize lifetime income, i.e., for these persons investing in education is never economically profitable. The opposite holds true for individuals with the highest ability $a = 1$, i.e. for them, investing in education is always economically profitable:

$$1 < \frac{(1 + \varepsilon_L)}{((1 - \lambda) + \varepsilon_H)\omega} < \frac{(1 + b)}{2}. \tag{3}$$

For the remainder of the paper, we assume that these conditions are fulfilled. Otherwise, either all individuals or no one would invest into education. If the relative wage changes, the threshold value changes according to:

$$\alpha'(\omega) = -\frac{(b - 1)(1 + \varepsilon_L)[(1 - \lambda) + \varepsilon_H]}{(b\omega[(1 - \lambda) + \varepsilon_H] - (1 + \varepsilon_L))^2} < 0. \tag{4}$$

A higher relative wage makes it favorable for agents with lower ability to invest in training. Even a small change in ω might result in a large reaction of α if the denominator of Equation (4) is close to zero.

Assumption 1: The gain in experience of high-skilled workers is at least as high as the gain in experience of low-skilled workers $\frac{\varepsilon_H}{(1-\lambda)} \geq \varepsilon_L$.

Proposition 1: Under Assumption 1, the first period low-skilled income of the marginal young individual with ability α who chooses to invest in further education is as least as high as the

institutions, e.g., colleges and universities. Alternatively, one could model the educational decision in a more elaborate way by relating the time to educate with the achievable stock of human capital and letting individuals choose the optimal time to educate. However, the basic qualitative results of our model would be the same.

income when working as a high-skilled worker.

Proof: The evaluation of the wage income of high-skilled and low-skilled workers within the respective age group at the ability threshold α using Equation (2) yields the stated result.

Corollar 1: The marginal student experiences a higher wage increase in absolute and relative terms when investing in training compared to always working as low-skilled: $(1+\alpha)(\varepsilon_L - 1) \leq (1+b\alpha)\,\omega\,(\varepsilon_H - (1-\lambda))$.

3 The wage distribution across age and skill groups

With workers differing in their age and skill level, we have four different wage distributions that together define the aggregate wage distribution in the economy. The wage income of young workers is distributed over the support $[1, \max\{1+\alpha, (1+b)(1-\lambda)\,\omega\}]$. According to Proposition 1, there is an overlap between the income distribution of low-skilled and high-skilled workers as the latter experience opportunity costs of education that are rewarded with higher income when old. The overlapping region ranges from $(1+b\alpha)(1-\lambda)\,\omega$ to $\min\{(1+\alpha), (1+b)(1-\lambda)\,\omega\}$. This is as it cannot be ruled out that the student with the highest ability $a = 1$ earns less than the marginal student who does not study. The distribution of young workers' wages is given by:

$$g_y(w) = \begin{cases} f\,(w-1) & w \in [1, (1+b\alpha)(1-\lambda)\omega) \\ f\,(w-1) + f\left(\frac{w}{(1-\lambda)\omega b} - \frac{1}{b}\right)\frac{1}{\omega b} & w \in [(1+b\alpha)(1-\lambda)\omega, (1+\alpha)) \\ f\left(\frac{w}{(1-\lambda)\omega b} - \frac{1}{b}\right)\frac{1}{\omega b} & w \in [(1+\alpha)\,, (1+b)\,(1-\lambda)\,\omega] \end{cases} \tag{5}$$

if $(1+\alpha) < (1+b)(1-\lambda)\,\omega$ and

$$g_y(w) = \begin{cases} f\,(w-1) & w \in [1, (1+b\alpha)(1-\lambda)\omega) \\ f\,(w-1) + f\left(\frac{w}{(1-\lambda)\omega b} - \frac{1}{b}\right)\frac{1}{\omega b} & w \in [(1+b\alpha)(1-\lambda)\omega, (1+b)\,(1-\lambda)\,\omega) \\ f\,(w-1) & w \in [(1+b)\,(1-\lambda)\,\omega, (1+\alpha)] \end{cases} \tag{6}$$

if $(1+\alpha) \geq (1+b)(1-\lambda)\,\omega$.

The mean wages of the young low-skilled and the young high-skilled workers can be defined as:

$$\overline{w}_{y,L}(\alpha) = 1 + \int_0^\alpha a\frac{f(a)}{F(\alpha)}da \tag{7}$$

and

$$\overline{w}_{y,H}(\alpha,\omega) = \left(1 + \frac{b}{1-F(\alpha)}\int_\alpha^1 af(a)da\right)\omega\,(1-\lambda)\,. \tag{8}$$

Any change in the relative wage ω directly and indirectly influences the wage distribution of the young: First, a relative wage increase widens the wage distribution by changing the support. Second, the relative wage changes the composition of both skill groups due to changing incentives to invest in education. The mean wage of all young workers is defined as:

$$\overline{w}_y(\alpha,\omega) = F(\alpha)\overline{w}_{y,L}(\alpha) + (1-F(\alpha))\,\overline{w}_{y,H}(\alpha,\omega). \tag{9}$$

A change in the relative wage changes the mean wage of the younger workers according to:

$$\frac{d\overline{w}_y(\alpha,\omega)}{d\omega} = [(1+\alpha) - (1+b\alpha)\,\omega\,(1-\lambda)]\,f(\alpha)\alpha'(\omega) + (1-F(\alpha))\,\frac{\overline{w}_{y,H}(\alpha,\omega)}{\omega}. \tag{10}$$

The change of the mean wage can be decomposed into the compositional effect and a direct wage effect. With Proposition 1, the compositional effect (first term) is strictly negative. In contrast, the direct effect (second term) is always positive. Therefore, an exogenous change of the relative wage ω is to a certain degree compensated by the endogenous compositional effect. That is, a higher relative wage induces more of the young to invest into their education. As a result, a part of the low-skilled wage income among the young is lost and replaced by a lower high-skilled wage income, which reduces the average wage of younger workers. The higher the density of the ability distribution at the extensive margin of education α, the more likely it is that the compositional effect over-compensates the relative wage increase, which eventually leads to a decline of the average wage of the young.

The wage income of the old generation is distributed on the support $[\varepsilon_L, (1+b)\,\varepsilon_H\omega]$ with the distribution function:

$$g_o(w) = \begin{cases} f\left(\frac{w-1}{\varepsilon_L}\right)\frac{1}{\varepsilon_L} & w \in [\varepsilon_L, (1+\alpha)\varepsilon_L], \\ f\left(\frac{w}{\varepsilon_H\omega b} - \frac{1}{b}\right)\frac{1}{\varepsilon_H\omega b} & w \in [(1+b\alpha)\varepsilon_H, (1+b)\,\varepsilon_H\omega]. \end{cases} \quad (11)$$

The mean wages of the old low-skilled and the old high-skilled workers are defined as:

$$\overline{w}_{o,L}(\alpha) = \left(1 + \int_0^\alpha a\frac{f(a)}{F(\alpha)}da\right)\varepsilon_L \quad (12)$$

and

$$\overline{w}_{o,H}(\alpha,\omega) = \left(1 + \frac{b}{1-F(\alpha)}\int_\alpha^1 af(a)da\right)\omega\varepsilon_H. \quad (13)$$

The mean wage of the older workers is given by:

$$\overline{w}_o(\alpha,\omega) = F(\alpha)\overline{w}_{o,L}(\alpha) + (1-F(\alpha))\,\overline{w}_{o,H}(\alpha,\omega) \quad (14)$$

and can be written in terms of the mean wage of the younger workers:

$$\overline{w}_o(\alpha,\omega) = F(\alpha)\overline{w}_{y,L}(\alpha)\varepsilon_L + (1-F(\alpha))\,\overline{w}_{y,H}(\alpha,\omega)\frac{\varepsilon_H}{(1-\lambda)} \quad (15)$$

In analogy to the young, the change of the mean wage of the old induced by a change in the relative wage can be computed as:

$$\frac{d\overline{w}_o(\alpha,\omega)}{d\omega} = [(1+\alpha)\,\varepsilon_L - (1+b\alpha)\,\omega\varepsilon_H]\,f(\alpha)\alpha'(\omega) + (1-F(\alpha))\frac{\overline{w}_{o,H}(\alpha,\omega)}{\omega} > 0. \quad (16)$$

In contrast to the young cohort, the compositional effect always magnifies the direct impact of the relative wage because the marginal worker with ability α earns more when working as a high-skilled worker instead of working as a low-skilled.

The aggregate wage distribution emerges as the mixture of both distributions of the young and the old with the combined support $[1, (1+b)\,\varepsilon_H\omega]$ weighted by the population share of each generation:

$$g(w) = \pi g_o(w) + (1-\pi)g_y(w). \quad (17)$$

The mean wage income in the economy is given by the weighted average of the mean wage income of the young and the old weighted with their population shares:

$$\overline{w}(\alpha(\omega),\omega) = \pi\overline{w}_o(\alpha(\omega),\omega) + (1-\pi)\,\overline{w}_y(\alpha(\omega),\omega) \quad (18)$$

To simplify the notation, we denote the support of the the wage distribution of the the young and old cohort by W_y and W_o, respectively. Since the mean wage of the younger cohort is strictly lower than that of the older cohort, an exogenous rise of the share of older workers always increases the mean wage. Calculating the change of the mean wage due to a change of the relative wage yields:

$$\frac{d\overline{w}(\alpha(\omega),\omega)}{d\omega} = \left[(1+\alpha)(\pi\varepsilon_L + (1-\pi)) - (1+b\alpha)\,\omega\,(\pi\varepsilon_H + (1-\pi)(1-\lambda))\right] f(\alpha)\alpha'(\omega) + (1-F(\alpha))\,\frac{\overline{w}_H(\alpha(\omega),\omega)}{\omega}. \tag{19}$$

Proposition 2: The average wage $\overline{w}$ is strictly increasing with the relative wage ω if the share of the old generation exceeds the share of the young, i.e., $\pi \geq 1/2$.

Proof: The sign of the derivative of the average wage with respect to the relative wage ω depends on the sign of the term

$$Z(\pi) = (1+\alpha)(\pi\varepsilon_L + (1-\pi)) - (1+b\alpha)\,\omega\,(\pi\varepsilon_H + (1-\pi)(1-\lambda)).$$

It holds that $Z(1/2) = 0$ while $Z(0) \geq 0$ and $Z(1) \leq 0$ because $\frac{\varepsilon_H}{1-\lambda} \geq \varepsilon_L$. Together with $Z'(\pi) \leq 0$, this yields the result of Proposition 2.

Overall, the compositional effect magnifies an average wage increase induced by an increase of the relative wage if the share of older worker is higher than the share of the younger workers. The higher the probability density is for the marginal student, the larger is the magnification effect. If instead the share of older workers is smaller than the share of the younger workers the compositional effect (partly) compensates the increase of the average wage.

3.1 Population aging and wage inequality

To measure wage inequality we use the mean log deviation (MLD). The MLD belongs to the class of general entropy measures which fulfill all axioms of decomposable inequality measures (Bourguignon 1979, Shorrocks 1980). The MLD is defined as follows:

$$MLD = \int_{w\in W} \log\left(\frac{\overline{w}(\cdot)}{w}\right) dG(w) \tag{20}$$

over the support of the wage distribution $w \in W$ where we dropped the functional arguments of the mean wage.[4] Since the logarithm is a concave function, the MLD is always positive. It aggregates the deviations from the mean of the distribution with more weight on wages below the average by taking the logarithm. Compared to other inequality measures, e.g., the Gini coefficient, the MLD is thus more sensitive to variations in the lower tail of a distribution.

As the wage distribution is the mixed distribution of the labor earnings of the young and old cohorts, we can decompose Equation (20):

$$MLD = \int_{w\in W_o} \log\left(\frac{\overline{w}(\cdot)}{w}\right) \pi dG_o(w) + \int_{w\in W_y} \log\left(\frac{\overline{w}(\cdot)}{w}\right) (1-\pi)\, dG_y(w). \tag{21}$$

One of the features of the MLD is that it is additively separable and can be used to decompose total cross-sectional inequality into inequality within separate subgroups of a population and

[4] In the remainder of the text we drop the functional arguments whenever it does not cause any confusion.

inequality between these subgroups, e.g., different worker groups:

$$MLD = \underbrace{\log\left(\frac{\overline{w}(\cdot)}{\overline{w}_o(\cdot)}\right)\pi + \log\left(\frac{\overline{w}(\cdot)}{\overline{w}_y(\cdot)}\right)(1-\pi)}_{\text{between}} + \underbrace{\pi MLD_o + (1-\pi)\,MLD_y}_{\text{within}}. \tag{22}$$

Accordingly, any change in the MLD can be attributed to changes of the between and/or the within component of inequality.

Our interest in the evolution of wage inequality is twofold: First, we want to disentangle the effect on inequality that stems from a change in the demographic structure of the work force. Second, we want to know how the change in the relative wage ω changes the composition of the different skill groups and how this influences wage inequality. To simplify the derivation of our results, we rewrite the MLD of the respective age group in terms of the ability distribution:

$$MLD_y = \log\left(\overline{w}_y(\cdot)\right) - \int_0^{\alpha} \log(1+a)\,dF(a) - \int_{\alpha}^{1} \log\left((1+ba)(1-\lambda)\,\omega\right) dF(a), \tag{23}$$

$$MLD_o = \log\left(\overline{w}_o(\cdot)\right) - \int_0^{\alpha} \log\left(\varepsilon_L(1+a)\right) dF(a) - \int_{\alpha}^{1} \log\left((1+ba)\,\varepsilon_H \omega\right) dF(a). \tag{24}$$

Comparing inequality among younger and older workers $MLD_o - MLD_y$ brings the following proposition:

Proposition 3: Wage inequality is higher among older workers than among younger workers $MLD_o > MLD_y$.

Proof: Calculation of the difference in the MLD between older and younger workers results in:

$$MLD_o - MLD_y = \log\left(\frac{\overline{w}_o(\cdot)}{\overline{w}_y(\cdot)}\right) - \log(\varepsilon_L)\,F(\alpha) - \log\left(\frac{\varepsilon_H}{1-\lambda}\right)(1-F(\alpha)).$$

Due to Equation (15), the ratio of mean wages between age groups is of the form

$$\frac{z\varepsilon_L + (1-z)\,\xi \frac{\varepsilon_H}{(1-\lambda)}}{z + (1-z)\,\xi} > 1 \quad \textit{with} \quad \xi > 1 \quad \textit{and} \quad z \in [0,1].$$

Define

$$h(z) = z\varepsilon_L + (1-z)\,\xi \frac{\varepsilon_H}{(1-\lambda)} \quad \textit{and} \quad \widetilde{h}(z) = (z + (1-z)\,\xi)\left(z\varepsilon_L + (1-z)\,\frac{\varepsilon_H}{(1-\lambda)}\right).$$

At the boundaries, it holds that $h(0) = \widetilde{h}(0)$ and $h(1) = \widetilde{h}(1)$ while $h(z) > \widetilde{h}(z)$ for $z \in (0,1)$ because $\widetilde{h}'(z), h'(z) < 0$ and $\widetilde{h}''(z) > 0$, $h''(z) = 0$. Substituting the lower bound function and applying Jensen's inequality results in

$$\log\left(\varepsilon_L F(\alpha) + \frac{\varepsilon_H}{1-\lambda}(1-F(\alpha))\right) \geq \log(\varepsilon_L)\,F(\alpha) - \log\left(\frac{\varepsilon_H}{1-\lambda}\right)(1-F(\alpha))$$

which proves that $MLD_o > MLD_y$.

How does the aggregate wage inequality change if the demographic structure changes? In order to answer this question, we differentiate the inequality measure with respect to π, the share of older workers:

$$\frac{\partial MLD}{\partial \pi} = \left[\frac{\overline{w}_o(\cdot) - \overline{w}_y(\cdot)}{\overline{w}(\cdot)} - \log\left(\frac{\overline{w}_o(\cdot)}{\overline{w}_y(\cdot)}\right)\right] + (MLD_o - MLD_y). \tag{25}$$

Within-group inequality, the second term, is positive and unambiguously contributes to a rise in aggregate inequality because the inequality among older workers is higher than the inequality among younger workers. Between-group inequality, the first term, is ambiguous and may contribute positively or negatively to overall wage inequality.

Proposition 4: In an economy with an aging society, i.e., with a shift in the demographic structure toward relatively more older workers ($d\pi > 0$), the change in wage inequality is magnified by between-group inequality if $\pi < \left(1/\log\left(\frac{\overline{w}_o(\cdot)}{\overline{w}_y(\cdot)}\right) - 1/\left(\frac{\overline{w}_o(\cdot)}{\overline{w}_y(\cdot)} - 1\right)\right)$ and (partly) compensated otherwise.

Proof: Solve the inequality $\frac{\overline{w}_o(\cdot)-\overline{w}_y(\cdot)}{\pi\overline{w}_o(\cdot)+(1-\pi)\overline{w}_y(\cdot)} - \log\left(\frac{\overline{w}_o(\cdot)}{\overline{w}_y(\cdot)}\right) > 0$ for π.

Therefore, demographic change (i.e., relatively more older workers) causes a rise in between-group inequality which magnifies the rise in within-group inequality as long as the share of older workers is not too high. This holds as long as to the productivity gaps between the young and the old of ε_L and $\varepsilon_H/(1-\lambda)$ for low-skilled and high-skilled workers, respectively, are not too large. That is, the differences in the population shares and the wage differences between the young and the old must be reasonable. However, even in the case of large differences, demographic change can still entail a rise in aggregate wage inequality since the reduction of between-group inequality may not be large enough to completely compensate the unambiguously positive contribution of within-group inequality.

3.2 Relative wage changes and wage inequality

The more direct way through which inequality is influenced is the relative wage channel:

$$\frac{dMLD}{d\omega} = \frac{\partial MLD}{\partial \omega} + \frac{\partial MLD}{\partial \alpha}\frac{d\alpha}{d\omega}. \tag{26}$$

It can be decomposed into the direct wage channel, the first term, and the indirect wage channel, the second term. A relative wage change directly impacts the wage distribution by increasing the dispersion of wages given educational choices. But with relative wages changing, agents revise their educational choices and thereby change the skill composition of the labor force which indirectly affects inequality. Both, between- and within-group inequality are affected through both channels. Foremost, a relative wage change induces more young individuals to invest into their education. Therefore, this indirect wage effect can be regarded as a rather long-run impact on inequality whereas the direct wage effect captures the short-run changes in inequality due to a relative wage change. The direct wage effect can be calculated as:

$$\frac{\partial MLD}{\partial \omega} = \underbrace{\frac{\frac{\partial \overline{w}}{\partial \omega}}{\overline{w}(\cdot)} - \left(\frac{\frac{\partial \overline{w}_o}{\partial \omega}}{\overline{w}_o(\cdot)}\pi + \frac{\frac{\partial \overline{w}_y}{\partial \omega}}{\overline{w}_y(\cdot)}(1-\pi)\right)}_{\text{between}} + \underbrace{\pi\left(\frac{\partial MLD_o}{\partial \omega}\right) + (1-\pi)\left(\frac{\partial MLD_y}{\partial \omega}\right)}_{\text{within}} \tag{27}$$

which after substituting Equation (19) and the derivatives of Equations (23) and (24) reduces to

$$\frac{\partial MLD}{\partial \omega} = \frac{(1-F(\alpha))}{\omega}\left(\frac{\overline{w}_H(\cdot)}{\overline{w}} - 1\right) > 0. \tag{28}$$

Therefore, the direct wage effect always increases wage inequality. The indirect wage channel can be calculated as:

$$\frac{\partial MLD}{\partial \alpha} = \frac{[(1+\alpha)(\pi\varepsilon_L + (1-\pi)) - (1+b\alpha)\,\omega\,(\pi\varepsilon_H + (1-\pi)(1-\lambda))]}{\overline{w}(\cdot)} + \left(\pi \log\left(\frac{\varepsilon_H}{\varepsilon_L(1-\lambda)}\right) + \log\left(\frac{1+b\alpha}{1+\alpha}\right) + \log\left(\frac{\omega}{1-\lambda}\right)\right). \tag{29}$$

Since $\frac{d\alpha}{d\omega} < 0$, the sign of $\frac{\partial MLD}{\partial \alpha}$ determines whether the indirect effect may (partly) compensate the direct wage effect. The last term of (29) is positive while the sign of the first term depends on the age distribution. Applying the result of Proposition 2, the first term is positive for any $\pi < 1/2$. Consequently, with a higher share of older workers the likelihood increases that $\frac{\partial MLD}{\partial \alpha} < 0$. If $\frac{\partial MLD}{\partial \alpha} < 0$ the indirect effect that works via a compositional change within the labor force supports the direct wage effect, i.e., inequality rises even more strongly.

In our stylized model we attribute a change in aggregate wage inequality, as measured by the MLD, either to changes in the relative wage ω or a change in the age structure of the economy represented by a change in π:

$$dMLD = \frac{\partial MLD}{\partial \omega} d\omega + \frac{\partial MLD}{\partial \alpha}\frac{\partial \alpha}{\partial \omega} d\omega + \frac{\partial MLD}{\partial \pi} d\pi. \tag{30}$$

In particular, the relative wage does not only affect wage inequality directly, but also indirectly by changing the incentives to educate and thereby altering the skill composition of the labor force. It is important to note that the latter endogenous change may also play a role whenever structural parameters of the economy change. This includes, e.g., changes in the educational system which increase the returns to higher education (i.e., a higher parameter b).

In order to study empirically to what extend changes in the demographic structure and the skill composition of the labor force contribute to the observed changes of wage inequality, we basically follow Equation (30) and decompose the changes in inequality along two dimensions, the age and skill composition of the labor force. In particular, we aim to disentangle empirically the changes in between- and within-group inequality from those brought about by compositional shifts from young to old, and low-skilled to high-skilled workers. According to our theoretical analysis, we expect that aging contributes positively to inequality. As does the compositional change of the skill structure. In addition, the compositional effects are expected to increase over time as the share of older workers rises.

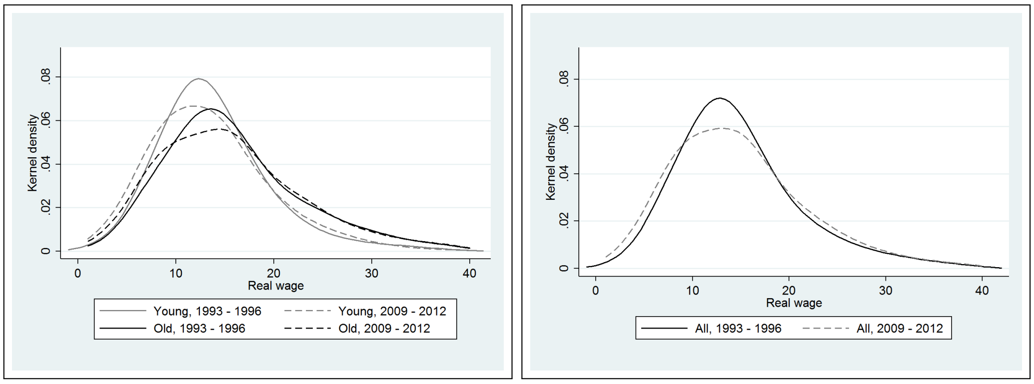
4 Data and empirical strategy

Our empirical analysis id based data from the German Socio-Economic Panel (SOEP). The rich individual-level data of the SOEP allow us to calculate hourly wages and to group individuals according to their age, education and other personal characteristics (for a detailed description of the SOEP, see Wagner et al. 2007). The use of cross-sectional and longitudinal weights makes the data and thereby the empirical results representative for Germany.[5]

Since we are interested in the development of the wage distribution, we limit our empirical analysis to the dependently employed population of working age, i.e., between the age of

[5] Whenever pooling data from more than one year, the individual weight in the first year when an individual is observed is the respective cross-sectional weight. In the following years, the weights for individuals who have been observed before are calculated by multiplying the weight from the preceding year with the respective longitudinal weight. The choice of the combination of cross-sectional and longitudinal weights or of cross-sectional weights alone have a negligible impact on our results.

FIGURE 1: Wage distribution by age and time



Notes: Density plots are based on a Gaussian kernel with 100 estimation points and a half-width of 2. Wages are in prices of 2010. For readability, wages higher than 40 EUR per hour are excluded. Data are weighted using the cross-sectional and longitudinal weights provided by the SOEP.
Source: Own calculations based on SOEP data

15 and 75. We exclude the self-employed (including helpers within family businesses) and individuals whose primary activity is not working, i.e., persons still in education (students, interns, apprentices) or in retirement. As the starting point, the years 1993 to 1996 are chosen. We leave out the years 1990 to 1992 because of the larger variability of the data in the aftermath following German reunification. The observations from several years are pooled in order to increase the sample size and limit the influence of possible outliers, thereby improving the accuracy of our estimates. The end point of the analysis is marked by the years 2009 to 2012.[6]

The individual wages are calculated from self-reported information about monthly gross labor earnings and hours worked. Essentially following Brenke and Müller (2013), we use the actual number of working hours if an individual reports to get paid for overtime work and the agreed number of working hours if overtime is not paid or if it is compensated via a personal working time account.[7] The average hourly wage amounts to roughly 16.50 Euro in the year 2012. Based on the bottom and top percentiles of the distribution, we exclude wages lower than 1 Euro or higher than 100 Euro as is common practice to limit measurement error.[8] All wage data are deflated to prices of 2010 with the consumer price index of the German Federal Statistical Office.

Our methodological approach to investigate empirically the theoretical prediction that compositional changes in the structure of the labor force account for a significant part of the observed change in wage inequality follows the decomposition of the MLD proposed by Shorrocks (1980) as used, e.g., by Peichl et al. (2012). For $k \in \{1, ..., K\}$ instead of only 2 population groups and applying the natural logarithm, Equation (22) becomes:

$$MLD = \frac{1}{n} \cdot \sum_{i=1}^{n} \ln\left(\frac{\overline{y}}{y_i}\right) = \underbrace{\sum_{k=1}^{K} \nu_k \cdot MLD_k}_{within} + \underbrace{\sum_{k=1}^{K} \nu_k \cdot \ln\left(\frac{\overline{y}}{\overline{y}_k}\right)}_{between}, \tag{31}$$

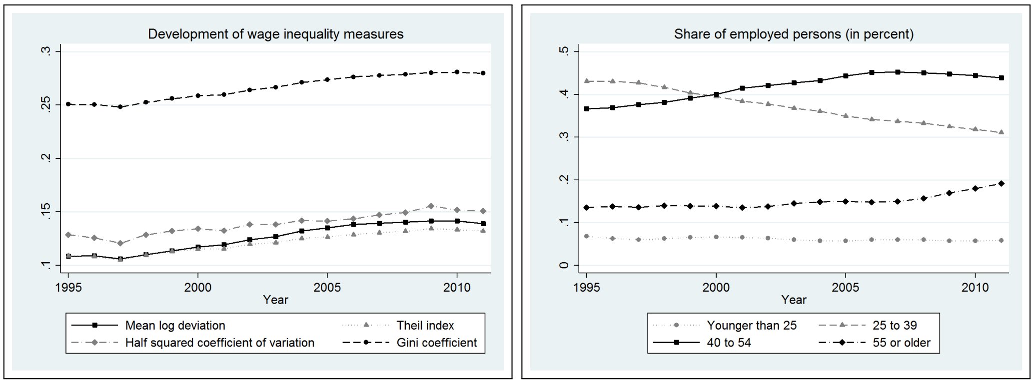
where $\overline{y}$ and y_i denote the population average income and the income for individual $i \in$

[6]The particular choice of the age bounds or the years pooled does not alter our results in any significant way (see Appendix).

[7]In case overtime is partly paid and partly unpaid, or in case of missing data, we employ the average of actual and agreed hours or whichever is available. The results are robust to using either actual or agreed hours only. For annual labor earnings and hours, the results diverge somewhat more in quantitative terms, but qualitatively still point into the same direction.

[8]In a robustness check, we also excluded wages lower than 3 Euro or higher than 50 Euro, as well as all observations from the SOEP's high income-sample without any substantial effect on the empirical results.

FIGURE 2: Development of age structure and wage inequality



Notes: Data weighted using the cross-sectional weights provided by the SOEP.
Source: Own calculations based on SOEP data

$\{1, ..., n\}$, respectively; ν_k represents the population share, MLD_k the mean log deviation and $\overline{y}_k$ the average income of group k. Figure 1 illustrates the differences in the wage distribution for two age groups. The older workers have higher average wages. In addition, older workers (here, 40 years or older) have more dispersed wages than their younger counterparts as argued in Proposition 3. As shown in Section 3, the overall wage distribution results as the weighted combination of the distinct group distributions. Over time, the group-specific and the overall wage distribution have become wider in Germany. The left panel of Figure 2 shows that the development of aggregate wage inequality turns out basically identical for different inequality measures.

For our baseline specification, we separate the labor force not only into the two groups "young" and "old", but into four age groups that mark particular segments over the working life cycle: under 25, 25 to under 40, 40 to under 55, 55 and older.[9] The first group comprises the labor market entrants from primary and secondary education and apprenticeships. In the second group, individuals with an academic education enter the labor force and the career paths start to diverge. The third group represents the core of the labor force marked by high participation rates across sexes and educational background. The last group of the older employees typically displays the largest heterogeneity because careers evolve differently and individual productivities diverge more strongly due to, e.g., health-related problems. Since individuals also start to exit the labor force, this group tends to be more selected.

Concerning the educational background, we define four groups based on the years spent on education: 10 years or less, 11 years, 12 years and 13 years or more.[10] Albeit not perfect, this classification creates four groups from low to high education and ensures that each group comprises at least 10% of the labor force. However, due to the particularities of the extensive German vocational training system, a classification based on years of education might not be completely adequate. Therefore, we also divide the labor force according to the CASMIN- and ISCED-classifications and vary the number of groups for all classifications. While the results for the compositional effects diverge more strongly for the educational than for the age classifications, they turn out largely robust when both characteristics are accounted for in combination. Only with the ISCED-classification, compositional effects are reduced in favor of between-group inequality.

With the MLD, the overall change in inequality over time can be separated into inequality

[9]We also separate the sample into 2, 3, 5 or 10 age groups and thereby reach the same conclusions. By construction, the share of within-group inequality in total inequality typically declines as the number of groups increases, and vice versa.

[10]Individuals with 10.5 (11.5) years of education are included in the groups with 11 (12) education years.

changes within and between population subgroups as well as compositional changes. Taking the discrete difference of Equation (31) between two time points t and t', a change in overall inequality measured by MLD can be decomposed into four terms (Mookherjee and Shorrocks 1982, Peichl et al. 2012):

$$\begin{aligned} \Delta MLD &= MLD_{t'} - MLD_t \\ &\approx \underbrace{\sum_{k=1}^{K} \overline{\nu}_k \cdot \Delta MLD_k}_{A} + \underbrace{\sum_{k=1}^{K} \overline{MLD}_k \cdot \Delta \nu_k}_{B} \\ &\quad + \underbrace{\sum_{k=1}^{K} [\overline{\lambda}_k - \overline{\ln(\lambda_k)}] \cdot \Delta \nu_k}_{C} + \underbrace{\sum_{k=1}^{K} (\overline{\theta}_k - \overline{\nu}_k) \cdot \Delta \ln(\overline{y}_k)}_{D}, \end{aligned} \tag{32}$$

where $\lambda_k = \overline{y}_k / \overline{y}$ denotes the relative average income of group k and $\theta_k = \nu_k \cdot \lambda_k$ expresses the share of group k in total income. Δ serves as the difference-operator between t' and t. A bar denotes averages over t' and t. The demographic shift within the labor force discussed in Section 3.1 must be considered rather slow. The right panel of Figure 2 shows the development of the employment shares of different age groups in Germany since the mid 1990's. In particular, it can be seen how the large cohort of the "Baby boomers" born at the beginning of the 1960's has grown older. The endogenous change in the skill composition due to higher educational investment induced by a change in relative wages as discussed in Section 3.2 also takes time. The time period of 16 years which we choose for our analysis seems sufficiently long for a study of composition effects.

The four terms A, B, C and D allow us to separate the influence of compositional (i.e., demographic or educational) changes on the wage distribution from changes in inequality brought about by other, e.g., technological or institutional, factors that affect the way certain characteristics are remunerated on the labor market. Of course, the compositional changes due to demographic and educational developments cannot be seen in isolation from other age- or cohort specific wage developments. Ceteris paribus, an increase in group size, i.e., labor supply, implies a downward pressure on wages and an upward pressure on inequality for this group. Therefore, our empirical analysis must be seen as purely descriptive, although it should give a rather conservative account of the importance of the compositional effects at work as long as the effect of cohort size on individual wages remains relatively unimportant compared to the influence of personal characteristics and overall economic developments.[11]

The first term A provides the change in total inequality attributable to within-group changes in inequality. By fixing the population shares of the groups at their average over time, this term reflects the changes in inequality due to, e.g., the technological, economic or institutional environment. Analogously, the term D gives the contribution of changes in the relative income positions of the different groups due to such factors.[12]

In contrast, the terms B and C account for the fact that compositional changes lead to changes in inequality even in the absence of any distributional effects within or between groups. Inequality can change because of shifts toward groups with more equal or unequal wage distributions (term C) and/or shifts toward groups with relatively low or high incomes compared to

[11] Cross-sectional wage regressions show stable or rather increasing returns to education and age over time. If at all, the relative increase in the supply of older and better educated employees dampens their relative wage increases compared to the younger and less educated.

[12] In term D, the change in a group's relative income is weighted with its average share in total income and its population share. Depending on the relation between income and population share, a rise in relative income leads to an increase (decrease) in inequality if a group is relatively rich (poor).

the population average (term D). For instance, an increase in the share of older employees is expected to increase wage inequality since this group displays both above-average wages and above-average inequality. The simultaneous decrease in the share of younger employees (who have below-average wages and inequality) might cause a decrease in total inequality at the same time, depending on whether the between- or the within-component prevails. The composition effects can partly offset each other (Section 3), and the overall "demographic effects" remain an empirical question. For illustrative purposes, the terms B and C are re-written as $\sum_{k=1}^{K}(\overline{MLD}_k - \overline{MLD}) \cdot \Delta\nu_k$ and $\sum_{k=1}^{K}[\overline{\lambda}_k - 1 - \overline{\ln(\lambda_k)}] \cdot \Delta\nu_k$, respectively.[13]

5 Results

Since the mid 1990's, overall wage inequality has increased by approximately 25% in Germany according to our calculations. As illustrated by Figure 2, the small share of employees under the age of 25 has declined to less than 6% in the years 2009 to 2012, the share of employees between age 25 and 39 has dropped from 44% in the years 1993 to 1996 to 32%. On the other hand, the share of workers 40 years or older has risen to 62%. In addition to the labor force shares, Table 1 reports the level and the changes of the age groups' average wages and inequality. The descriptive statistics highlight the differences between groups and provide some first evidence for the importance of compositional changes. While the average real wage has decreased within each age group over time, overall it has remained stable due to the shift from younger employees with relatively low wages to higher paid older employees. As argued in Proposition 3, wage inequality is higher for older than for younger employees. With regard to its development, inequality has increased most strongly among the old and the young, but much less for those between age 40 and 55.

The decomposition results for the wage inequality development by age groups are given in Table 2. Overall, wage inequality has increased by 25% between 1993/96 to 2009/12. With almost 20%-points, inequality changes within groups account for the largest fraction of the inequality increase. However, even the crude breakdown of the labor force into four age brackets already attributes more than 12% of the overall change to compositional changes. The analysis also reveals the various changes at the group level that underlie the aggregate developments. The largest within-group changes in wage inequality (column A) are observed for the 25 to 39 year-olds and those of age 55 and older. Between age groups (column D), the marked decrease in real wages of the youngest group has contributed 10% to the overall inequality increase. There is no sign that the relatively scarcity of the young pushes up wages for these employees.

With regard to the composition effects, only the increase of the share of the oldest group has an unambiguously positive contribution to aggregate inequality because this group has both above-average wages and above-average inequality. For the other age groups, the changes in group size translate into partly compensating compositional effects depending on relative wages (column B) and relative inequality (column C): The decrease of the share of the two younger cohorts supports an increase in inequality because of their relative homogeneous wage structures and, at the same time, a decrease in inequality because of their relatively low wages. This pattern is reversed for the group of the 40 to 54 year-olds, which comprises the

[13]The subtraction of the overall means of $\overline{MLD}$, $\overline{\lambda} = 1$ and $\overline{\ln\lambda} = 0$ serves to illustrate better how the changes in the population share of a group k affect inequality. For instance, a relative decline of the younger cohorts (with a relatively low level of inequality) increases inequality. But by construction, the corresponding summand in Equation (32) would show a negative contribution because it does not take into account how the inequality in group k relates to overall inequality. This distinction is irrelevant for the contribution of compositional effects at the aggregate level.

Table 1: Descriptive statistics by age groups, 1993–96 to 2009–12

	Population share		Real wage		MLD	
	1993–1996	**Δ**	**1993–1996**	**Δ**	**1993–1996**	**Δ**
Total	100.00	0.00	15.25	-0.00	10.97	2.77
	(0.00)	(0.00)	(0.07)	(0.10)	(0.18)	(0.33)
Younger than 25	7.11	-1.45	10.69	-1.37	8.46	2.40
	(0.20)	(0.29)	(0.13)	(0.21)	(0.55)	(0.89)
25 to 39	44.35	-12.26	14.74	-0.26	9.30	1.89
	(0.40)	(0.71)	(0.10)	(0.18)	(0.28)	(0.60)
40 to 54	35.58	8.36	16.41	-0.14	11.24	0.95
	(0.39)	(0.65)	(0.12)	(0.15)	(0.27)	(0.38)
55 and older	12.96	5.35	16.31	-0.34	13.09	5.72
	(0.27)	(0.45)	(0.24)	(0.32)	(0.63)	(0.94)

Notes: Table reports mean values of population shares (in %), real wages (in Euros of 2010) and the mean log deviation (MLD), bootstrapped standard errors in parentheses (500 replications). Δ refers to the difference between 1993–96 and 2009–12. Due to some missing values, the results can vary to a small degree across specifications and Tables. All data are weighted using the cross-sectional and longitudinal weights of the SOEP.
Source: Own calculations based on SOEP data

Table 2: Wage inequality decomposition by age groups, 1993–96 to 2009–12

	ΔMLD	**A**	**B**	**C**	**D**	$\frac{B+C}{\Delta MLD}$
Total	25.32	19.61	3.98	-0.89	2.57	12.37
	(3.25)	(3.18)	(0.50)	(0.22)	(0.54)	(2.47)
Younger than 25		1.40	0.36	-1.05	2.71	-2.80
		(0.52)	(0.09)	(0.22)	(0.44)	(0.77)
25 to 39		6.61	2.36	-0.10	0.25	9.06
		(2.14)	(0.30)	(0.03)	(0.18)	(1.60)
40 to 54		3.45	-0.49	0.19	-0.22	-1.22
		(1.41)	(0.14)	(0.02)	(0.24)	(0.56)
55 and older		8.16	1.75	0.08	-0.17	7.33
		(1.40)	(0.24)	(0.03)	(0.16)	(1.21)

Notes: The total change of MLD (first column) and the share thereof due to compositional changes (last column) are given in percent. All results for A, B, C and D are given in percentage points. Bootstrapped standard errors (500 replications) are reported in parentheses. Due to some missing values, the results can vary to a small degree across specifications and Tables. Data are weighted using the cross-sectional and longitudinal weights of the SOEP.
Source: Own calculations based on SOEP data

large "Baby Boomer"-cohort in Germany. In total, the relative increase of this age group has contributed slightly negatively to aggregate inequality due to a less dispersed wage structure. In sum, the results support Proposition 4 that a decline in between-group inequality partly compensates the shift toward higher within-group inequality.

When splitting the labor force according to the educational background in Table 3, it shows that the labor force has become more skilled from 1993/96 to 2009/12. The share of employees with at least 12 years of education has increased from 51% to 67%. The groups with less education have experienced the strongest declines in real wages and the largest increases in group inequality which mirrors that educational demands have become more important over time. As for the breakdown by age, the importance of compositional changes is reflected in the wage developments: While the real wage has declined for each group, there is a slight increase overall because of the shift toward the group with most education years who have significantly higher wages.[14] The relative wage of the high compared to the low skilled has gone up considerably.

The decomposition results for education are reported in Table 4. Approximately 9% of the overall change in inequality can be attributed to compositional changes. In particular, the increase in the share of employees with 13 or more years of education and relatively high wages has exerted an upward pressure on wage inequality which was partly offset by the decline of employees with 10 or less education years with lower wages. For different educational

[14]Due to some missing values for the educational variables, the descriptive statistics and decomposition results differ to a small extent across the empirical specifications. This has no consequences for any of the conclusions drawn from the analysis.

TABLE 3: Descriptive statistics by education groups, 1993–96 to 2009–12

	Population share		Real wage		MLD	
	1993–1996	**Δ**	**1993–1996**	**Δ**	**1993–1996**	**Δ**
Total	100.00	0.00	15.27	0.10	10.99	2.65
	(0.00)	(0.00)	(0.07)	(0.11)	(0.18)	(0.33)
10 years or less	15.53	-5.95	12.36	-1.10	9.54	3.84
	(0.27)	(0.39)	(0.13)	(0.23)	(0.52)	(1.03)
11 years	33.19	-9.88	14.45	-0.68	8.11	3.75
	(0.38)	(0.59)	(0.09)	(0.17)	(0.27)	(0.61)
12 years	27.05	6.42	14.33	-0.52	10.49	0.74
	(0.37)	(0.61)	(0.12)	(0.16)	(0.35)	(0.50)
13 years or more	24.22	9.41	19.30	-0.11	11.72	0.78
	(0.38)	(0.60)	(0.18)	(0.24)	(0.36)	(0.54)

Notes: Table reports mean values of population shares (in %), real wages (in Euros of 2010) and the mean log deviation (MLD), bootstrapped standard errors in parentheses (500 replications). Δ refers to the difference between 1993–96 and 2009–12. Due to some missing values, the results can vary to a small degree across specifications and Tables. All data are weighted using the cross-sectional and longitudinal weights of the SOEP.
Source: Own calculations based on SOEP data

TABLE 4: Wage inequality decomposition by education groups, 1993–96 to 2009–12

	ΔMLD	**A**	**B**	**C**	**D**	$\frac{B+C}{\Delta MLD}$
Total	24.14	18.14	1.53	0.56	3.76	8.83
	(3.29)	(3.07)	(0.47)	(0.21)	(1.09)	(2.51)
10 years or less		4.39	0.47	-1.75	2.35	-5.37
		(1.19)	(0.28)	(0.18)	(0.51)	(1.22)
11 years		9.65	2.10	-0.33	0.92	7.47
		(1.61)	(0.25)	(0.05)	(0.26)	(1.39)
12 years		2.05	-0.85	0.22	0.86	-2.66
		(1.38)	(0.15)	(0.03)	(0.26)	(0.65)
13 years or more		2.06	-0.18	2.42	-0.37	9.40
		(1.44)	(0.18)	(0.18)	(0.84)	(1.61)

Notes: The total change of MLD (first column) and the share thereof due to compositional changes (last column) are given in percent. All results for A, B, C and D are given in percentage points. Bootstrapped standard errors (500 replications) are reported in parentheses. Due to some missing values, the results can vary to a small degree across specifications and Tables. Data are weighted using the cross-sectional and longitudinal weights of the SOEP.
Source: Own calculations based on SOEP data

groupings, the contribution of the compositional effects to the overall change in inequality varies between approximately 2% for the ISCED-classification and 13% for only 2 groups based on education years (see Appendix Table 6).

Since the age and education classifications might partly overlap, we split the labor force into 16 groups according to both characteristics. By construction, the explanatory contribution of the within-group inequality changes (part A) declines with a higher granularity of the population. Table 5 shows that, while the importance of between-group inequality increases, the compositional effects now account for a much larger fraction of the inequality increase of up to 25%.[15] The largest within-group inequality contributions are found for the oldest worker groups and those with 11 years of education. The only group whose wage inequality has declined are the 40 to 54 years-olds with the highest educational background. In line with the findings from the separate decompositions for age and education, the shifts away from the relative homogeneous group of the 25 to 39 year-olds and toward the high wage-earners with 13 or more years of education have pushed up wage inequality. As argued in Section 3.2, the changes in the skill composition magnify the wage inequality increase via within- and between-group inequality.

As a methodological robustness check, we follow the procedure of Lemieux (2006) and estimate separate wage regressions for the time periods 1993 to 1996 and 2009 to 2012 with age and age squared interacted with the educational variable. In order to account for the compositional

[15]For different groupings according to education years or the CASMIN-classification, the compositional effects make up 20% to 25%. Based on the ISCED-classification, the compositional effects turn out smaller at 11% to 15%.

TABLE 5: Wage inequality decomposition by age and education groups, 1993–96 to 2009–12

Education	Age	ΔMLD	A	B	C	D	$\frac{B+C}{\Delta MLD}$
Total		24.14	14.62	4.83	1.10	3.56	25.00
		(3.29)	(2.95)	(0.66)	(0.44)	(1.15)	(4.44)
10 years or less	Younger than 25		0.31	-0.02	-1.32	1.22	-5.66
			(0.34)	(0.15)	(0.21)	(0.29)	(1.36)
	25 to 39		0.37	0.72	-0.90	1.69	-0.74
			(0.52)	(0.19)	(0.14)	(0.21)	(0.82)
	40 to 54		1.31	0.18	-0.20	0.09	-0.10
			(0.51)	(0.07)	(0.05)	(0.21)	(0.27)
	55 and older		1.97	-0.05	-0.28	-0.00	-1.36
			(0.74)	(0.13)	(0.08)	(0.29)	(0.58)
11 years	Younger than 25		0.41	0.32	-0.45	0.61	-0.55
			(0.24)	(0.10)	(0.10)	(0.16)	(0.46)
	25 to 39		2.48	2.33	-0.65	0.88	7.09
			(0.71)	(0.31)	(0.13)	(0.20)	(1.62)
	40 to 54		0.87	1.19	-0.06	0.21	4.76
			(0.64)	(0.17)	(0.02)	(0.10)	(0.84)
	55 and older		3.76	0.11	0.01	0.11	0.50
			(1.01)	(0.09)	(0.01)	(0.10)	(0.37)
12 years	Younger than 25		0.42	0.34	-0.58	0.38	-1.02
			(0.28)	(0.09)	(0.11)	(0.22)	(0.40)
	25 to 39		0.03	1.28	-0.42	0.35	3.63
			(0.81)	(0.18)	(0.06)	(0.28)	(0.75)
	40 to 54		0.15	-1.23	0.17	0.17	-4.47
			(0.75)	(0.27)	(0.05)	(0.12)	(1.31)
	55 and older		1.29	0.30	0.22	0.03	2.22
			(0.61)	(0.24)	(0.10)	(0.16)	(1.31)
13 years or more	Younger than 25		0.09	-0.09	0.33	0.28	1.02
			(0.23)	(0.06)	(0.11)	(0.19)	(0.41)
	25 to 39		0.36	-0.24	0.08	-0.22	-0.68
			(1.24)	(0.12)	(0.04)	(0.35)	(0.40)
	40 to 54		-0.73	-0.89	3.11	-1.17	9.38
			(0.63)	(0.16)	(0.27)	(0.65)	(1.72)
	55 and older		1.53	0.58	2.02	-1.07	10.97
			(0.57)	(0.19)	(0.24)	(0.50)	(1.86)

Notes: The total change of MLD (first column) and the share thereof due to compositional changes (last column) are given in percent. All results for A, B, C and D are given in percentage points. Bootstrapped standard errors (500 replications) are reported in parentheses. Due to some missing values, the results can vary to a small degree across specifications and Tables. Data are weighted using the cross-sectional and longitudinal weights of the SOEP.
Source: Own calculations based on SOEP data

changes, first, a logit model is estimated for the pooled sample of both time periods with a dummy variable indicating either the start or the end period and the same explanatory variables as in the wage regression. The predicted probabilities from these regressions are used to re-weight the observations.[16] The contribution of labor force composition changes to the overall change in inequality are given by the difference between the change in residual wage inequality actually observed and the counterfactual change in residual wage inequality when the data are re-weighted. Depending on the choice of either the start or the end period as the base year, and the educational indicator (education years, CASMIN- or ISCED-classification in greater detail), the compositional effects account for 19% to 28%.

6 Discussion and conclusion

The public discussions about wage and income inequality typically take place at a very aggregated level and are centered around the observed development of some aggregate measure of inequality, e.g., the Gini coefficient. Often, an increase in measured inequality is equated with economically or socially unfair developments that call for policy interventions. However, movements in aggregate inequality indices alone cannot support such claims. On the one hand, there exists no "optimal" degree of inequality which could serve as a benchmark for the assessment of current inequality. Consequently, it is also impossible to qualify a change in

[16]The term "re-weighted" reflect that the data are weighted with the cross-sectional of the SOEP in order to make the data representative for Germany in a given year and (re-)weighted with the probabilities of being observed in a particular time period in order to account for the compositional differences.

measured inequality – a priori – as good or bad. On the other hand, changes in inequality can come about in many different ways. In the analysis at hand, we highlight the importance of compositional shifts that call for a quite different interpretation of inequality changes than those caused by, e.g., technological progress or institutional changes.

Wage regressions that link individual wages to selected explanatory characteristics are a straight-forward way to investigate wage and inequality developments more closely. Some of these developments turn out "explainable" in empirical studies and can be traced back to personal, institutional or workplace characteristics. For instance, it is now widely accepted that technological change and increased global competition can partly explain why real wages have increased much less at the bottom of the wage distribution than at the top since the 1980's. However, a large part of the labor market and wage developments remains empirically unexplainable. The discussions about inequality often focus on this residual wage inequality. As Lemieux (2006) points out, the unexplained inequality changes can comprise at least three very distinct phenomena: (i) changes in the way unobservable characteristics are rewarded on the labor market, (ii) changes in the composition of the labor force that entail a changing distribution of unobservable skills, or (iii) plain measurement error.

Obviously, the three different reasons lead to very different interpretations regarding an observed change in aggregate inequality measures. In particular, the mechanical effects of compositional changes within the labor force do not necessarily call for a policy intervention. In fact, a rise in wage inequality can be the logic and intended consequence of demographic or institutional developments. In Germany, the main factors that can be expected to support a more dispersed wage distribution must be seen in the demographic changes, a general increase in educational attainment and the labor market reforms in the first half of the 2000's. Compared to the labor market situation in the 1990's, there has been a shift toward older employees who dispose of more experience and thus, on average, achieve higher wages. Due to the heterogeneity and the uncertainty surrounding individual career paths, the wage distribution is also more dispersed for older than for younger employees. In addition, the strong increase in the labor market participation of females and older individuals has contributed to a more diverse labor force which is mirrored in the inequality measures.

With regard to the educational background of the labor force, the increase in university graduates also contributes to a rise in wage dispersion because this particular group typically has higher and less evenly distributed wages. In addition, the labor market reforms supported a stronger integration of the unemployed into the labor market. Since this group comprises individuals with lower (observable and unobservable) skills, a rise in wage inequality must be seen as the natural result of an institutional reform that aims at increasing the labor market participation of low-qualified individuals. In sum, the current labor force is composed very differently than 10 or 20 years ago. The interaction between a high labor market participation across many population groups and the corresponding wage and inequality developments presents a fruitful area for further research. For instance, the integration of the large baby-boomer cohort or the low-qualified unemployed into employment most likely requires an increase in wage dispersion.

As theoretically predicted, we find that compositional shifts in the age and educational structure of the labor force account for a significant share of up to a quarter of the observed change in aggregate wage inequality in Germany since the mid 1990's. The importance of the compositional effects appears somewhat larger in our analysis than in previous studies, e.g., Gernandt and Pfeiffer (2007), Dustmann et al. (2009), Antonczyk et al. (2010). This may be attributed to the longer observation period which here includes seven years after the labor market reforms. In addition, the demographic shifts toward older workers takes place gradually which suggests that the implied changes in inequality take time to materialize. The same applies to the endogenous changes in the skill structure brought about by changes in the relative wage and the corresponding changes in the returns to educational. The theoretical analysis suggests that a decline in the number of young low-skilled workers (who typically earn lower wages) due to a higher number of young workers investing in their education exerts an inequality-reducing effect as long as the share of the young does not fall beneath a certain threshold. Once the share of the older workers is relatively large, inequality starts to rise with more educational investment because more workers move into the group of the high skilled which is marked by high within-group inequality while the reduction of inequality between age and skill groups becomes smaller.

From a policy perspective, our theoretical and empirical analysis highlights the importance of understanding the mechanisms that can cause a change in inequality indices. An increase in measured inequality is neither a necessary nor a sufficient condition for an economically or socially worrying development. In fact, a rise in inequality might even mirror an increase in economic and societal prosperity. In general, increasing diversity of the labor force and a more dispersed wage distribution can be seen as a central feature of a modern economy and society. Therefore, it is important to ask the right questions: How much of the observed distributional changes can be explained empirically, how much remains unexplained? How can the explained and unexplained developments be interpreted best? Do they call for policy interventions or do they rather mirror the logical or intended consequences of broader economic, societal or institutional developments? A well-founded evidence-based policy addresses these questions as detailed as possible.

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Appendix – Additional Tables

Table 6: Wage inequality decomposition by education and age groups, 1993–96 to 2009–12

Specification	ΔMLD	**A**	**B**	**C**	**D**	$\frac{B+C}{\Delta MLD}$
Education only						
Education years (2 groups)	24.14	18.68	1.18	1.88	2.35	12.92
	(3.29)	(3.02)	(0.27)	(0.14)	(1.03)	(2.20)
Education years (3 groups)	24.14	18.76	0.91	0.55	3.79	6.17
	(3.29)	(3.03)	(0.38)	(0.20)	(1.09)	(2.01)
CASMIN (3 categories)	24.50	19.23	0.52	1.53	3.13	8.52
	(3.28)	(3.11)	(0.42)	(0.23)	(1.05)	(2.19)
CASMIN (4 categories)	24.50	18.83	0.10	0.63	4.75	3.00
	(3.28)	(3.07)	(0.41)	(0.25)	(1.10)	(2.03)
ISCED (3 categories)	24.17	18.40	-0.10	0.49	5.24	1.64
	(3.25)	(2.98)	(0.37)	(0.14)	(1.05)	(1.68)
ISCED (4 categories)	24.17	18.48	-0.69	1.11	5.13	1.75
	(3.25)	(2.96)	(0.37)	(0.23)	(1.12)	(1.87)
Age and education						
Age and education years (2 groups)	24.14	14.58	3.83	2.00	3.51	24.55
	(3.29)	(2.94)	(0.53)	(0.40)	(1.11)	(3.89)
Age and education years (3 groups)	24.14	15.27	3.60	0.94	3.93	19.10
	(3.29)	(2.93)	(0.58)	(0.42)	(1.15)	(3.60)
Age and education (CASMIN, 3 categories)	24.50	14.88	4.25	0.55	4.85	19.86
	(3.28)	(2.96)	(0.65)	(0.42)	(1.11)	(3.86)
Age and education (CASMIN, 4 categories)	24.50	14.54	4.38	0.38	5.25	19.69
	(3.28)	(2.92)	(0.67)	(0.43)	(1.13)	(3.87)
Age and education (ISCED, 3 categories)	24.17	14.45	3.23	0.25	5.88	14.60
	(3.25)	(2.89)	(0.57)	(0.34)	(1.12)	(3.13)
Age and education (ISCED, 4 categories)	24.17	14.26	2.43	0.31	6.80	11.49
	(3.25)	(2.85)	(0.58)	(0.39)	(1.16)	(2.96)

Notes: The total change of MLD (first column) and the share thereof due to compositional changes (last column) are given in percent. All results for A, B, C and D are given in percentage points. Bootstrapped standard errors (500 replications) are reported in parentheses. Due to some missing values, the results can vary to a small degree across specifications and Tables. Data are weighted using the cross-sectional and longitudinal weights of the SOEP.
Source: Own calculations based on SOEP data

Table 7: Wage inequality decomposition by age groups, 1993–96 to 2009–12 (Different age groups and time periods)

Specification	ΔMLD	**A**	**B**	**C**	**D**	$\frac{B+C}{\Delta MLD}$
2 age groups (<40; >=40)	25.32	22.38	2.41	-0.22	0.73	8.79
	(3.25)	(3.35)	(0.45)	(0.04)	(0.49)	(2.09)
3 age groups (<35; 35-55; >55)	25.32	21.68	2.97	-0.85	1.48	8.51
	(3.25)	(3.41)	(0.51)	(0.11)	(0.69)	(2.16)
4 age groups (Core working age: 20 to 60 years)	22.42	17.80	2.56	-1.26	3.23	5.93
	(3.19)	(3.10)	(0.40)	(0.19)	(0.75)	(2.14)
5 age groups (<25; <35; <45; <55; >55)	25.32	20.24	3.51	-1.12	2.65	9.60
	(3.25)	(3.28)	(0.51)	(0.22)	(0.60)	(2.41)
10 age groups (<20 to >60)	25.32	17.65	5.58	-1.40	3.47	16.76
	(3.25)	(3.12)	(0.59)	(0.24)	(0.63)	(3.19)
Years: 1992-95 to 2009-12	20.12	14.19	4.09	-1.28	3.03	14.25
	(3.09)	(3.02)	(0.47)	(0.21)	(0.55)	(3.29)
Years: 1996-99 to 2009-12	28.43	23.78	3.31	-0.07	1.42	11.52
	(3.27)	(3.20)	(0.49)	(0.24)	(0.58)	(2.17)
Years: 1999-2002 to 2009-12	15.73	12.31	2.45	-0.41	1.38	13.45
	(3.05)	(2.95)	(0.33)	(0.22)	(0.55)	(3.68)
Years: 1993-96 to 2003-06	21.41	18.26	1.97	-0.92	2.06	4.97
	(2.65)	(2.52)	(0.28)	(0.18)	(0.52)	(1.55)

Notes: The total change of MLD (first column) and the share thereof due to compositional changes (last column) are given in percent. All results for A, B, C and D are given in percentage points. Bootstrapped standard errors (500 replications) are reported in parentheses. Due to some missing values, the results can vary to a small degree across specifications and Tables. Data are weighted using the cross-sectional and longitudinal weights of the SOEP.
Source: Own calculations based on SOEP data

Table 8: Wage inequality decomposition by age groups, 1993–96 to 2009–12 (Robustness checks w.r.t. wage calculation and weighting procedure)

Specification	ΔMLD	**A**	**B**	**C**	**D**	$\frac{B+C}{\Delta MLD}$
Wage based on agreed hours	23.56	18.71	3.89	-1.23	2.14	11.47
	(3.08)	(2.93)	(0.41)	(0.24)	(0.61)	(2.36)
Wage based on actual hours	26.49	21.22	3.66	-0.87	2.44	10.71
	(3.29)	(3.19)	(0.51)	(0.23)	(0.57)	(2.46)
Wage based on annual data	23.72	23.35	2.20	-0.48	-1.38	7.37
	(2.97)	(3.07)	(0.40)	(0.14)	(0.76)	(2.02)
Real wage based on GSOEP CPI	26.99	21.17	4.02	-0.90	2.65	11.71
	(3.31)	(3.23)	(0.51)	(0.22)	(0.55)	(2.30)
Nominal wage	24.72	19.07	3.96	-0.89	2.53	12.60
	(3.22)	(3.14)	(0.50)	(0.22)	(0.54)	(2.52)
Weights including first waves	24.39	18.87	3.99	-1.14	2.61	11.86
	(3.06)	(2.99)	(0.47)	(0.20)	(0.49)	(2.43)
Cross-sect. weights only	25.94	20.07	3.69	-0.76	2.89	11.39
	(3.08)	(2.92)	(0.44)	(0.21)	(0.53)	(1.99)
Unweighted	44.99	33.52	6.29	-2.60	7.06	8.21
	(2.20)	(2.11)	(0.39)	(0.18)	(0.42)	(1.03)
Excl. high-income sample	22.37	17.60	3.52	-0.80	2.03	12.39
	(2.99)	(2.96)	(0.47)	(0.21)	(0.53)	(2.82)
Excl. very low and very high wages	22.17	16.48	4.07	-0.97	2.55	14.08
	(2.20)	(2.04)	(0.36)	(0.23)	(0.59)	(2.13)

Notes: The total change of MLD (first column) and the share thereof due to compositional changes (last column) are given in percent. All results for A, B, C and D are given in percentage points. Bootstrapped standard errors (500 replications) are reported in parentheses. Due to some missing values, the results can vary to a small degree across specifications and Tables. Data are weighted using the cross-sectional and longitudinal weights of the SOEP.
Source: Own calculations based on SOEP data